fundamental particle like a proton or an electron. If we wish to know its position, we must observe its effect in scattering light or material particles. But since the experiment involves the exchange of energy in one way or another between the colliding particles, we are unable to determine the momentum of the original particle. In a similar way, measurements of momentum do not allow an accurate simultaneous determination of position. The more accurately we try to determine the one, by using, for example, light of a shorter wave length in the first experiment, the less accurate becomes our determination of the other. The result is that we can specify position and velocity simultaneously only in terms of probability.[50]

This principle of indeterminism, one of the foundation-stones of modern quantum mechanics, thus imposes a limit on observability—a limit that lies in the very nature of experiment. In classical mechanics, it was thought that if we knew the state of a physical system at a given instant, its state at any future instant could be accurately predicted by mechanical laws.[51] In quantum

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[50] Objective indeterminism makes no sense. It is obvious that unless objectively there is something stable and determined with respect to which the indeterminism is said to exist, we could never even recognize or enunciate the principle of indeterminacy. Unless this determinate something is objectively real, the indeterminism described by the Heisenberg principle (which amounts to inaccuracy in measurement) could never be real either. The ontic implications of the Heisenberg principle is an argument for the reality of what traditional philosophy calls "form," "substance," and "nature."

[51] Laplace wrote: "We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated, and the respective situation of the human beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be more uncertain, and the future and the past would be present to its eyes. The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence. Its discovery in mechanics and geometry, added to that of universal gravity, have enabled it to comprehend in the same analytical expressions the past and the future states of the system of the world. Applying the same method to some other objects of its knowledge, it has succeeded in referring to

mechanics, only probable predictions can be made because of the Heisenberg principle which enforces limits on our knowledge.[52] As contemporary physicists are accustomed to say, the law of causality no longer applies in nature. But obviously causality does not have the same meaning for the physicist as for the philosopher. For the latter, it means entitative dependence; for the former it means the predictability of events from a knowledge of their previous states.[53] Causation, for the physicist, means the order of nature, and it is beginning to assume the meaning of our knowledge of that order. This raises the question of whether indeterminacy is subjective or objective.

In their popular works, men like Eddington and Jeans[54] have insisted that the principle is objective. Heisenberg also holds this opinion because, he says, the restoration of determinism would destroy quantum mechanics.[55] Opinion on this point must be reserved by such thinkers as the logical positivists. Their so-called physicalism forces all questions which are outside the scope of scientific method, into the category of the meaningless.

On the other hand, Planck and Einstein have insisted that indeterminism is only subjective, an inexactitude in our knowledge

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general laws observed phenomena and in foreseeing those which given circumstances ought to produce," *Philosophical Essay on Probability,* New York, 1902, p. 4.

[52] There are really two indeterminisms in modern physics: 1) That caused by the interaction between instrument and system; and 2) That caused by a lapse of time (considered quantum-mechanically) between the interference and the recording of the result. Cf. Tolman, R. C., *The Principles of Statistical Mechanics,* Oxford, 1938, p. 416 and p. 232.

[53] "I propose," writes Planck, "to commence the next stage with the simple and general proposition that an event is causally conditioned if it can be foretold with certainty," *Where Is Science Going?,* p. 45.

[54] Eddington, for instance, argued that religion became possible for the man of science after 1927, the year of the formulation of the Heisenberg principle. Probability equations in modern physics have been evoked to prove free will, to refute mechanistic biology, to explain evolution, and Sommerfeld has even argued that the soul is present illocally in the body, just as a corpuscle is associated with a wave in some indeterminate way. Cf. Frank, P., *Interpretations and Misinterpretations of Modern Physics,* Paris, 1938.

[55] *Op. cit.,* pp. 12-13.

of nature but not an indeterminism in nature itself, a temporary barrier but not a permanent limit.

The fact that physicists have not always kept in mind the distinction between the logical and ontological orders has considerably obscured their attitude toward the physical significance of the principle. To ask for the simultaneous specification of the position and momentum of a particle is, they hold, a meaningless question: therefore, there is in nature an inherent natural limit to measurement (equal at least to Planck's constant h). If a phenomenon is not measurable, it simply does not exist for physics, they add; the conclusion is that there is position without momentum and momentum without position in the real order.[56]

In interpreting the principle of uncertainty, Planck and Schroedinger have pointed out that the question of its status has been asked in the wrong way. It is not an empirical question to be settled by measurement. This would only impale it in a vicious circle. For so long as men rely on measurements to settle the question, the very conditions of the problem preclude the answer.[57] We can resolve the question only on the premise that there is a legitimate philosophical approach to matter in addition to the method of science. A question may be meaningless to the physicist with his method of mensuration and yet be very meaningful to the

[56] Pauli makes a distinction between indeterminism and our own ignorance. "Die Unterscheidung zwischen (prinzipieller) *Unbestimmtheit* und *Unbekanntheit* sind für die ganze Quantentheorie entscheidend. Dies möge erläutert werden am Beispeil einer Versuchsanordnung, bei welcher ein Lichtquant die Möglichkeit hat durch zwei Löcher zu treten und auf einem dahinterliegenden Schirm (im statischen Mittel bei oftmaliger Wiederhölung des Versuches) eine Beugungsfigur zu erzeugen. In diesem Fall ist es unbestimmt durch welches Loch das Lichtquant geflogen ist. Wenn dagegen eine Versuchsanordnung vorliegt bei der sicher nur ein Loch für das Lichtquant geöffnet ist, dann sagen wir es ist nicht bekannt, durch welches Loch das Lichtquant geflogen ist." "Die Allgemeine Prinzipien der Wellenmechanik," *Handbuch der Physik*, vol. 24, pp. 88-9. However, one may still ask why the path of the photon should be indetermined in the first case and unknown in the second. If we could measure their paths, both would turn out to be ontally determined. Indeterminism is a logical limit; it is not an ontal dislocation.

[57] Schroedinger, *op. cit.*, p. 116; Planck, *Wege zur physikalischen Erkenntnis*, p. 163.

philosopher searching after the more remote characteristics of matter by intellectual analysis. If the scientific method is to be the sole instrument for valid knowledge, then we shall continue to confound the order of being with the order of knowing and wind up in utter skepticism. Scientific method cannot even decide the question of the objectivity of knowledge. If it alone can exhaust reality, then reality is equated with the metrical aspects of matter. Posed from the philosophical viewpoint, the question of indeterminism is synonymous with the problem of nature's order. We shall return to this question in Chapter IV.

The problem of indeterminism is a pseudo-problem. It has arisen from an equivocation in the meaning of the word "determinism."[58] As Bertrand Russell says, "The Principle of Indeterminacy has to do with measurement not with causation (the order of nature)."[59] It is not an ontal fact. It is an epistemological limit of measurement. To substitute the latter for the former is as anthropomorphic as the nineteenth-century naiveté which contemporary scientists want to do away with in their science.

But if contempoprary quantum mechanics has been forced to abandon the search for an exact formulation of law in individual cases, it has nevertheless adopted a substitute expression of order and uniformity without which true science would be impossible. It is the substitute of statistics. By this means, physics seeks for ontal order at a lower level.[60]

In the present chapter, it has been noted that measurement is included in the point of view taken by physics on its subject-matter. The purpose of measurement is to give a numerical, not ontological, representation of reality. Measurement is limited ontally by the fact that it cannot resolve quality as quality into its forms. It is limited logically by being an approximation and by being subject to the principle of indeterminism. The mathematical character of measurement may now be discussed.

[58] Stebbing, L. S., *A Modern Introduction to Logic*, London, 1933, pp. 183-4.

[59] *The Scientific Outlook*, New York, 1931, p. 105.

[60] Maritain, J., *op. cit.*, p. 53.

CHAPTER III

Applied Mathematics

Statistics, so true in general, so fallacious in particular, as Gibbon said, is a way of studying the average fate over a large array, of realities which we cannot measure individually and which can be evaluated in individual cases only in terms of probability.

Though statistical mechanics has received peculiar stress with the advent of quantum theory which now rules the field in the study of atomic and subatomic particles, it was known and used in classical physics where the knowledge of the initial condition of an ensemble was inexact; where precise individual measurements (of molecules for example) were impossible for empirical reasons; and where exact computation was impracticable because of the large numbers involved.

Laplace, Poisson, and Bernoulli worked out theories of probability in mathematical terms. Maxwell and Boltzmann developed and applied statistics to phenomena of the kinetic-molecular theory. It was Maxwell's equations that were applied in such problems as the determination of the mean free path of molecules in a volume of gas and their average velocity. Boltzmann's outstanding contribution to classical physics was his mathematical interpretation of the second law of thermodynamics. His H-theorem, which calculates the tendency of molecules to approach equilibrium distribution, enables us to forecast with reasonable exactness the future behavior of a system of particles when our knowledge of their original state is incompletely specified.[1]

Before the advent of quantum mechanics, classical physics had also applied statistical methods to the study of radioactive decay. In phenomena of radioactivity, alpha-, beta-, and gamma-particles

[1] Maxwell had investigated the "probability distribution" of gases before Boltzmann's fundamental work was published. Hence the physical equation for determining the most probable state of a gas through statistical means is called the Maxwell-Boltzmann distribution law.

are liberated from atoms with the result that the radiating atoms of one element break down with time into simpler elements by a type of atom-smashing in the world of nature itself. By a simple mathematical equation, the time of decay of a definite fraction of an aggregate of atoms can be precisely computed, if the number of atoms present is known in advance. The time at which our analysis begins seems to have no effect in determining the fate of the aggregate. Half of an aggregate of radon atoms will break up within 3.825 days after our study is commenced, or they will survive the period; RaA has a half-life[2] of three minutes, RaB of 28.8 minutes, and so on. But though we cannot determine the fate of the single atom, we can, in dealing with large numbers, arrive at a very simple mathematical relation stating the half-life period of the aggregate. This precise calculation over large aggregates of atoms indicates that though the decay of the individual atom seems at first sight to be a random process, there is in reality an order that we have not been able to grasp.

In quantum mechanics, statistics are employed not only because of the practical difficulties that made it easier for classical physics to deal with large aggregates on a statistical basis. There is an additional theoretical difficulty in the Heisenberg principle whereby we can only state the probability-amplitude of an individual particle without being able simultaneously to specify its position and momentum.[3] The Schroedinger equation, which is fundamental to contemporary study of quantum phenomena, yields a statement of the most probable positions of a particle.[4] Applied to increasingly large numbers of particles, it can specify values with increasing accuracy. Finally, it becomes identical with the determinism of classical mechanics according to the Bohr correspondence principle.

Statistical mechanics has been especially fruitful in the study of thermodynamics. By virtue of its principles, we eliminate in a sense the idea of heat since we deal only with the potential

[2] Half-life period is the time required for half of the radioactive atoms present at a given moment to decay.

[3] Cf. *supra*, pp. 37 ff.

[4] Cf. Mayer and Mayer, *Statistical Mechanics*, New York, 1940, pp. 43 ff.

and kinetic energy of atoms themselves.[5] Applied to the second law of thermodynamics, statistics has led to the opinion that entropy has only a macroscopic meaning and that the behavior of individual systems of an ensemble is a tendency toward disorder in which only a probable prediction of future phenomena is possible. Because of this disordered movement which is amenable only to statistical treatment, a pail of water might conceivably freeze on a fire, it is said, instead of evaporating; or a stone might "fall upward."[6] Of course, such an opinion willingly admits that the odds are overwhelmingly against such wild possibilities. But having generalized the statistical analysis of macroscopic thermodynamic phenomena, it believes justified in viewing all rigidly entropic processes as a tendency toward disorder in which anything might happen.

The application of statistics is not a mere exercise in mathematics. It demands analysis on the part of the scientist to collect and classify data and to select, through judgments of relevance and irrelevance, those factors which are essential to a statistical treatment of a collection and those which are not.[7] Thus, contemporary physics has three statistical ways of treating particle phenomena. If there are n distinguishable particles, a modified Maxwell-Boltzmann distribution law can be applied. In the case of n indistinguishable particles with symmetric functions[8]—nuclei and atoms having an even number of particles and also photons treated as particles—the Einstein-Bose statistics can be applied. In the case of n indistinguishable particles subject to anti-symmetric restrictions—electrons, protons, neutrons, or nuclei and atoms having an odd number of such particles—the Fermi-Dirac distribution laws can be used.[9] Although the Boltzmann

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[5] Bridgman, P. W., *The Nature of Thermodynamics*, Cambridge (Mass.), 1940, p. 100.

[6] *Ibid.*, pp. 162-164.

[7] Keynes, J. M., *Treatise on Probability*, London, 1921.

[8] A function is symmetric when the interchange of two fundamental particles does not involve changing the sign of the function. It is anti-symmetric when such an exchange results in the changing of the sign.

[9] Tolman, *op. cit.*, p. 367.

statistics is mathematically simpler than the others, it does not take into account the mechanical interaction of the particles.[10]

The widespread use of statistics in modern physics, especially since the advent of the Heisenberg principle, has led to the belief that all laws are merely statistical laws. But such a view is confuted by philosophical analysis. If there is an order in nature, if chance is, as Bacon said, the mark of our own ignorance rather than of nature's blunders, then statistics is merely a way of treating aggregates of phenomena, the individual cases of which we are unable, because of experimental difficulties or because of the large numbers of systems, to treat with the mathematical rigor that we use on the macroscopic world. Probability-amplitudes and apparent chance relationships are applied on the level of the individual only because of the experimental or computational difficulties. But from another point of view, statistics is an exact science like any other—with its own subject-matter, its own axioms, and its own conclusions.

Laplace defined probability as the ratio of favorable cases to unfavorable ones. In a more specific sense, this means that by analysis of a set of data over a suitable range we can arrive at a limiting recurring rate for a particular sample and calculate, by integrating the equation thus found, the distribution of the sample in the whole collective. It is apparent that once this limiting frequency value has been established, the proportions in the end-result do not change no matter how large the aggregate becomes.

The three important statistical theories in vogue in modern physics—the Boltzmann, Einstein-Bose, and Fermi-Dirac systems—start out by an assumption concerning the initial distribution of the particles. Boltzmann, supposing that molecules were absolutely elastic and spherical bodies—a concept that was rightly criticized by such men as Stallo and Mach—made the assumption that certain ranges of the states of the molecules were equally probable. Though the quantum statistics rejects Boltzmann's billiard-ball concept of particles, they are at one with him in certain fundamental conceptions concerning equal probabilities. The Einstein-Bose theory, instead of considering individual particles,

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[10] Mayer and Mayer, *op. cit.*, pp. 63 ff.

assumes that each distribution of the whole is equally likely. The Fermi-Dirac system is founded on the assumption that a distribution has equal probabilities only when all the particles have different energies and are in different states.[11] But the assumptions concerning the initial distributions which underlie each of the three theories are not wholly without contact in the real world. The computations to which they each give rise can be tested by experimental measurements. All three systems have proved verifiable within their respective domains of application.[12] It is a case, in the language of de Morgan, of using experiments not to draw theories from but to test theories by. The test justifies in some sense the original assumptions and shows that the theory as a whole, in both its physical presuppositions and its mathematical development, has somehow come into real contact with the real world. The average is not a complete artefact, even though it may not be in 1-to-1 correspondence with reality.

Statistics has been contrasted with the so-called causal laws of physics which enable us to make accurate and not merely probable predictions. But this distinction cannot be maintained on closer analysis. Statistics is a science, like any other mathematical (or physico-mathematical) discipline. It differs from the other mathematical sciences not in its accuracy but in its subject-matter.[13] We are ignorant of the state of individual systems. Hence we begin with *averages* in the initial distribution. From this, however, we can accurately forecast the *averages* in the end-distribution. In view of this *exact* correlation, the individuals which form the average have somehow been obeying order and law. What statistics does is to reinforce the so-called principle of causation. This principle is not an empirical problem. The final answer as to whether nature is ordered or not must be based on intellectual analysis.

Indeed, statistics is a case for the principle of nature's uniformity in two ways: Firstly, if we start off with only probable prem-

[11] de Broglie, L., *Matière et Lumière*, Paris, 1937, p. 255.
[12] v. Mises, R., *Wahrscheinlichkeit, Statistik, und Wahrheit*, Vienna, 1936, pp. 240-242; Keynes, *op. cit.*, p. 391.
[13] v. Mises, *op. cit.*, p. 35.

ises, we can attain only a probable conclusion. Therefore, there is a form of determinism between our point of departure and our end-result.[14] Probability does not arise from the unpredictability, in individual cases, of nature in itself. It comes from our ignorance of the premises on which actual prediction must be based. When we cannot predict the future state of a system from knowledge of its previous state, this is so only because we cannot know the previous state. But since probable premises, in individual cases, can yield probable conclusions and since the probability can be exactly stated, we have not abandoned the determinism of the individual but sought it, as Maritain remarks, at a lower level. Secondly, over large aggregates, though we disregard the fate of the individuals in an aggregate, determinism reappears in the convergence of accurate predictions toward the limit of one; quantum mechanics approaches classical mechanics according to the correspondence principle. These two facts seem to argue against the views of certain modern physicists that because of our ignorance of nature, as shown in statistics, we must abandon objective determinism and equate scientific measurements with the whole course of nature. Such a view is a most literal anthropomorphism, of the type which Protagoras espoused, when he wrote that man's mind is the measure of all things.

A similar answer must eventually be invoked against the problems raised by the statistical treatment of thermodynamics. The so-called disorder is only an apparent one, a disorder in knowledge rather than of being. The ordered obedience of macroscopic thermodynamical phenomena to the law of entropy is proof of this fact. Statistically, we cannot speak of entropic individuals in thermodynamics since entropy does not exist on the level of the individual by definition. In the domain where heat exists, entropy holds. Statistics may explain away heat by reducing it to kinetic and potential energy. We may still wonder whether this statistical treatment really exhausts the subject. The macroscopic tendency of matter to produce unusable energy is still a challenging fact. The skepticism towards all law on the basis of the statistical treatment of thermodynamics is grounded on the homocentric belief

[14] Tolman, *op. cit.*, p. 210.

that we have proved the lack of order in nature simply by our failure to detect that order, the belief that the apparent disorder is a real one, that out of the disorder may arise in chance form such ordered happenings as the freezing of water on a fire or the falling of a stone upward. The disorder of increasing entropy is in reality only an order of a different type from that of reversible processes; this is shown by the ordered macroscopic phenomena which result and which prove deeply below the surface that the individual particles are obeying an order which eludes us but which is none the less real, ineluctable, and compelling for all that. We cannot simply conclude from the complexities in understanding or measuring a thing that it does not exist. If the universe is ordered as a whole and in its parts, disorder is no more the natural effect of order than chance can be the natural effect of purpose.[15]

Probability is only a mental state. It is not a sort of excluded-middle which reigns in the objective world. In themselves, relations affirmed by judgment are either true or false. When the evidence is not sufficient for us to attain certitude with regard to their truth or falsity, we may then arrive at a problematic judgment based on lesser evidence. As Plato put it, truth is to opinion as being is to becoming.

The distinction must be made between the empirical and logical treatment of probability. It is perfectly within the scope of physics, and of statistics in general, to express its probabilities in mathematical terms. Mathematics is the exact language demanded by empirical science. But mathematics is not logic. The mathematics of probability is a mathematical system based on numerical analysis. It can neither support demonstrative logic nor contradict it. It is on a different level. Yet outside of the domain of statistics, there is a logic of probability which differs both from apodictic reasoning and from the mathematics of probability. If it be asked whether Shakespeare had a long formal education, whether another war is likely within twenty-five years, or whether Plato wrote

[15] "If this law (the second law of thermodynamics) governed world development, God would have created a most improbable initial constellation which, left to itself, could only deterioriate into a growing disorder of increasing probability," Riezler, *op. cit.*, p. 16.

the dialogues in the order suggested by Lutoslawski, only a probable answer can be given—an answer which is outside the scope of mathematical expression but which is nevertheless a perfectly legitimate region of knowledge. Newman developed a logic based on convergent probabilities. Aristotle himself devotes a whole section of his organon, the *Topics,* to dialectical or probable argument. Statistics, as far as it obeys mathematical laws, is an exact science, although errors can easily arise in interpretation within the field where statistics are applied. When the physicist goes beyond his figures to say that matter is indetermined, he does what he says he should never do, namely, he interprets.

The foregoing discussion has already indicated the fact that the results of physical measurement are expressed in mathematical form. Mathematics is the language of modern physics. If physics deals with matter in terms of its quantitative aspects, it is readily understandable that mathematics, the science of quantity, should have a fundamental role in empirical study. Aquinas points out that the quantity of a being is known through measurement and that measurement is based on unity and number.[16] After Descartes had made quantity the essence of a body and mathematized the philosophic study of the universe, mathematics deposed philosophy as the *scientia rectrix* of empirical knowledge. But philosophical-minded physicists have failed to appreciate the epistemological significance of mathematical physics, with the result that the science of measurable reality has tended to identify reality with its measurable aspects. Mathematical entities in physics are a stenographic method of expressing physical concepts. Strictly speaking, the information contained or implied in a differential equation or "world" geometry could be stated in physical language.[17] But mathematics is brief, systematic, and exact. The brevity of an equation as compared with the cumbersome form of ordinary language is self-evident. The systematic interrelatedness of the various entities of a mathematical system enables us to reason with great ease and economy.[18] Finally, the symbolic character of mathematical entities

[16] *In Met.* 10, 2 (*passim*).
[17] Dantzig, T., *Number, the Language of Science,* New York, 1933, p. 233.
[18] Dantzig, T., *Aspects of Science,* New York, 1937, p. 75.

makes for a greater exactitude than would be possible, in the language of sense-intuition, when dealing with the metrical aspects of matter. It is this "objective" representation of experience in number-form which has led physicists to believe that rigorous attention to the mathematical symbolism strips their science of its anthropomorphic character.[19]

Mathematics is a science of its own right. It is indeed a perfect type of science, a *scientia propter quid,* in which the reasons for the attribution of a predicate to a subject are seen within the subject itself.[20] But by its subject-matter, it is circumscribed. It is the study of idealized quantity. As a *scientia propter quid,* it can demonstrate why one quantity (or aspect of quantity) is contained within another. It cannot perform the same function in the world of real being. Quantity is not co-terminous with reality. It is in fact the lowest kind of being. In the real world, we always have a quantity of *something.* What that something is escapes the grasp of mathematics. If mathematics is a *scientia propter quid,* it detaches quantity from its accidental status in the ontal world. It considers quantity as an independent, substantial entity for purposes of study. It deduces its various relationships. Not concerned with existential reality, truth in mathematics is not the conformity of thought with thing but of thought with thought—the legitimate sphere of the coherence theory of truth. On this premise, we can understand how there can be Euclidian, Riemannian, Lobatschevskian, and *n* geometries; how mathematically they can all be valid. In the physical sphere, it is another and perhaps insoluble question to ask whether space is Euclidian or not. In pure mathematics, space may be defined in any way the mathematician chooses so long as he does not, in his system, violate the principle of contradiction.[21]

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[19] Nunn, T. P., *Anthropomorphism and Physics,* London, 1928, p. 8.
[20] Maritain, *op. cit.,* pp. 82-3.
[21] "Mathematical achievement shall be measured by standards which are peculiar to the mathematicians. These standards are independent of the crude reality of our senses. They are: freedom from logical contradiction, the generality of laws governing the created form, the kinship which exists between this new form and those which have preceded it," Dantzig, *Number, the Language of Science,* p. 231.

This description applies only to the mathematics, which came to maturity, and perhaps even into childhood, during modern times. Such modern mathematics—whether its foundations be called *logical,* as in the works of Whitehead and Russell, or *empirical,* as in the works of a host of modern thinkers—is inductive in character; inclined to emphasize geometry as more fundamental than the science of number; and in general disposed to substitute for the concept of unity the notion of mathematical infinity which found fruitful applications only in the last several centuries. Classical mathematics is inclined to emphasize that each number is a different *species* rather than the limit of a continuous series. It is only this modern mathematics which is to be gauged solely by the principle of consistency.

The problem is to be explored much further than present limits allow. Its solution, it would appear, is not crucial for developments in the following pages which, when referring to mathematics, naturally emphasize the modern inductive type displayed by mathematical physics.

The idealized (abstract) character of mathematical entities prescinds from the idea of actuality. As employed, however, in physics, mathematics is no longer pure but applied.[22] It is then no longer a science of idealized forms as such. It seeks to bring the world into conformity with its abstract entities. Here lies the approximate character of physics that was already outlined in discussing the limits of measurement. Extended reality does not obey abstract mathematics which deals with perfect spheres, whole numbers, holomorphic functions, and other idealized entities.[23] To obtain continuous curves in the establishment of a relationship like that expressed in Boyle's law, we must interpolate values. We must content ourselves in measurement with a great but not the greatest number of decimal points and often with orders of magnitude without being able to specify the figures within those orders. Though we may be working with discrete elements, we consider

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[22] Cohen, M. R., *Reason and Nature,* New York, 1932, pp. 173 ff. Einstein (*Geometrie und Erfahrung,* Berlin, 1921, p. 8), makes a distinction between axiomatic and practical geometry.
[23] d'Abro, A., *Evolution of Scientific Thought from Newton to Einstein,* p. 34.

them to form a continuum so that the powerful tool of mathematical analysis (the calculus) may be applied. In short, the physical world can only approximate mathematical abstractions. As Einstein says, "As far as the statements of mathematics relate to reality, they are not certain; and as far as they are certain, they do not relate to reality."[24]

Whitehead has worked out a theory to bridge the gap between the physical and mathematical and thus to make possible the application of mathematical measurements to a physical reality. It is the method of "extensive abstraction." Mathematics in its pure form, Whitehead argues, does away with spatial characteristics. It treats them as inextended points. It does away with temporal characteristics, treating them as durationless instants. The problem is one of reconciling the point-moment mathematical scheme and other such constructions with which science deals and the sensible, extended, enduring reality with which it begins.

To solve the problem, Whitehead proposes that we consider an intellectual movement of convergence to simplicity in which quantities such as volume or duration are allowed to grow smaller and smaller, approaching—but never reaching—an ideal limit. If we are measuring volume, we can, Whitehead states, conceive of a nest of Chinese boxes. These unfold one after another in the analogy. There is an indefinite sequence with each box being smaller than the one before it. Though the series may tend toward an ideal limit, the quantitative expression never reaches it. The smallest box still has volume. It is always extended by whatever small amount.[25] Whitehead compares his method to the procedure of the calculus "in the region of numerical calculation, namely, it converts a process of approximation into an instrument of exact thought."[26] However small we let our element of volume decrease, it is always extended by an amount, however small. Time calculation can be treated in a simular way.[27] By retaining the physical

[24] Einstein, *op. cit.*, pp. 103-4.
[25] *Concept of Nature*, pp. 80-81.
[26] *Principles of Natural Knowledge*, p. 76.
[27] Whitehead sums up his method of extensive abstraction thus: "A set of events is called an 'abstractive class' when (i) of any two of its members

characteristics of—to use Whitehead's term—an event, however small it becomes, the method of extensive abstraction aims to provide a logical basis for the application of mathematics to physical data. Because of its similarity to the theory of the calculus, a likeness which Whitehead himself avows, the method of "extensive abstraction" has struck responsive chords in physicists who see in it a way of keeping their science on a physical basis.

But Whitehead has only restated the problem. He has not solved it. The problem that attracts him can be solved in terms of the traditional notion of abstraction. In mathematics, the mind prescinds from all sensible attributes of bodies except their quantity. It considers this in the abstract apart from its physical existence. In returning to matter, man's intellect applies its mathematical reasoning. It must recognize in the applications of numbers and lines that it can only approximate mathematical entities in the physical world. Whitehead's method cannot substitute for this traditional notion of abstraction without which mathematics cannot be explained.

The mind must work with the "ideal limit" to study its characters and to derive their laws. The line between this "limit" and the members of the convergent series is not continuous, to borrow a term from mathematics. There is a discretum, a gap. It divides mathematics from mathematical physics, exactitude from approximation, the abstract from the concrete, and between these couples there is no middle term. Whitehead's method does not bridge the gap. His problem can only find a realistic treatment if we bear

one extends over the other, and (ii) there is no event which is extended over by every other event of the set.

"The properties of an abstractive class secure that its members form a series in which the predecessors extend over their successors, and that the extension of the members of the series (as we pass towards the 'converging end,' comprising the smaller numbers) diminishes without limit; so that there is no end to the series in this direction along it and the diminution of the extension finally excludes any assignable event. Thus any property of the individual events which survives throughout members of the series as we pass towards the converging end is a property belonging to an ideal simplicity which is beyond that of any one assignable event. There is no event which the series marks out, but the series itself is a route of approximation towards an ideal simplicity of 'content,'" *ibid.*, p. 104.

in mind the division of mathematical and physical sciences based upon the degrees of abstraction. Mathematics is one thing. Physics is quite another. When we apply to the physical world the idealized forms of mathematics, the result is an approximation. The mathematical is not an ideal case of the physical in the sense of calculus where *ideal* means *limit.* It is an ideal of the physical in the sense of traditional abstraction. In the method of "extensive abstraction," mathematics is not differentiated from the physical sciences. The difference which Whitehead proposed is one which "excludes any assignable" distinguishing event, a terminology which Whitehead adopts from the calculus. He transfers the methods of calculus from their legitimate sphere to that of intellectual operations which are no more pure quantity than quantity is pure spirit. Whitehead, here as in other cases, merely restates problems which he sets out to solve.

Mathematics is not the plenary explanation of reality. Bertrand Russell remarks, "Physics is mathematical, not because we know so much about the physical world but because we know so little; it is only its mathematical properties that we can discover."[28] Even in mathematical physics, mathematics must be supplemented by the non-mathematical if its statements are to have any meaning in the physical world. Facts are not proved by a mathematical equation except in a derivative sense, when other facts have been previously supplied by experiment. Mathematics affords a way of describing experiments and observation, reasoning about them, and generalizing these conclusions. It cannot take their place.[29] "Mathematics is neither a description of nature nor an explanation of its operation," writes Boyer. "It is not concerned with physical motion or with the metaphysical generation of quantities. It is merely the symbolic logic of possible relations, and as such is concerned with neither approximate nor absolute truth. That is, mathematics determines what conclusions will follow logically from given premises."[30]

[28] *Philosophy,* New York, 1927, p. 157. The intellectualist will not agree with Russell's view that the empirical method is the only approach to reality.
[29] Thompson, W. R., *Science and Common Sense,* New York, 1937, p. 124.
[30] Boyer, C., *The Concept of the Calculus,* New York, 1939, p. 308.

In present-day physics, the physical basis of mathematical formulae is oftentimes neglected both by preference and by practice. The new mathematical physics has, we are told, forced us to abandon such constructs as models and mechanical theories and to substitute a schema of mathematical equations. The result is that physics is becoming more and more mathematical and less and less capable of tracing its mathematical complexities to any real counterparts in the physical world. This growingly abstract character of physics, though it has obvious advantages, is a definite danger to the noetic stability of the science. As we ascend from the world of sense experience into the stratosphere of mathematical abstractions, physics may awaken to find that it has lost contact with the real world which it aims to study and explain. Physics, if it is to remain physics, cannot substitute the mathematical world for the data of experiment. Without radiation, Bohr's formula for the hydrogen spectrum would be a mere arithmetical fiction. Without the law of gravitation, the equations of dynamics in classical physics would have remained only a source of mathematical exercises.[31] "Mathematics," Herzfeld notes, "is only a tool, a shorthand way of expression, but cannot add anything to the physical concept, although it might occasionally suggest a physical law because its mathematical expression might be particularly simple."[32] Tool here is equivalent to formal object.

[31] Dantzig, T., *Aspects of Science,* p. 251.
[32] "The Frontiers of Modern Physics and Philosophy," p. 40.

CHAPTER IV

Empirical Law

Law in physics is not merely a mathematical equation. It is a physico-mathematical equation, a physical relation stated in mathematical form. Since Mill's time, the term "law" has been used to express a uniformity of connection between phenomena.

In the strict sense, "law," as applied to the irrational world is a misnomer. Aquinas noted that the word stems from the Latin "ligare" meaning "to bind."[1] Only rational creatures can be obligated by a rule or plan. The irrational world has no duties.[2] It is only by analogy, as Aquinas states, that the concept of law can be applied to irrational creatures.[3] He viewed law as a rule or measure of operations, according to which an agent is led to act or not to act.[4] He pointed out that every creature was subject to law, since, not being the author of its own being, it is not the author of its own purpose.[5] He viewed creatures as endowed with natural tendencies to seek their ends.[6] Law is an expression of natural tendency.

Final causes are in a sense the primary causes in nature: without a fore-determined end for the activity of creatures, there would not be a sufficient reason why one effect should flow from a given action rather than another.[7] It was these natural tendencies of creatures toward their proper ends that Aquinas called laws of nature.[8] In their very essence then is the notion of end, the striving of being to perfect itself according to the intentions carved into

[1] I-II, 90, 1.
[2] *Cont. Gent.* III, 104.
[3] I-II, 91, 2.
[4] I-II, 90, 1.
[5] I-II, 5, 6.
[6] I-II, 84, 3.
[7] *Cont. Gent.* III, 2.
[8] *In De Div. Nom.* X, 1.

its nature. The schoolmen did not seek mechanical laws of matter in a special science. They rendered matter intelligible in the only way that it can be so rendered. If man and matter are without purpose and goal by nature, then whim can, and should, rule the world and decide its destiny. The schoolmen, seeking God's laws in the things He made, found material being insufficient unto itself. They regarded it in the light of purpose and finality. Such notions the scientific method cannot touch. Yet they remain as challenging facts to thinking men.

Law always implies finality. It is a directive plan of action toward an end.[9] In human action, law is apprehended by the agent himself, who directs his own nature toward its end. In the subhuman world, cognition is likewise necessary for law. Not being possible, however, in the mineral, plant, and animal world, it exists in the mind of God who knows the ends of the subintelligent agents and directs these beings toward their preconceived goals. In the philosophical sense, laws are therefore tendencies in the order of finality.

In physics, where finality, not being susceptible to quantitative measurement, is quite naturally not considered, law has come to mean something different. It is a generalization of the quantitative correlations which are empirically observed and measured. Facts and laws are thus not radically distinct.[10] Laws, in general are a universalization of facts. Our language bears witness to this relationship. We are inclined to speak of the laws themselves as facts and to say, for example, "it is a fact that force equals mass times acceleration" or "it is a fact that electric current equals electromotive force over resistance."[11] "When people say that they appeal to facts they usually appeal to well-established laws," as

[9] I-II, 93, 3.
[10] Lalande, A., *Theórie de l'Induction et de l'Expérimentation,* Paris, 1929, p. 5.
[11] Cohen and Nagel, *op. cit.,* pp. 217-8, distinguish four meanings of the word "fact": a) Discriminated elements in sense perception like the lines of a spectrum or pointer-readings which are referred to as signs of facts rather than the facts themselves; b) Particular propositions stating a relation observed between *this thing* and *that thing* but not going beyond actual here-and-now observation; and c) General propositions, universalizing the conclusions of b; and d) The existence of a correlation between things existing in space and time.

Ritchie notes, "but they often think that they mean an appeal to sense-perception, to particular events."[12] In the stricter sense, the relations instanced above are laws. They generalize the measured associations found in experiment. Campbell argues that such qualitative statements as the description of the properties of substances in chemically identifying the elements, for example, are also laws.[13] But scientists do not refer to such phenomena as "laws." Law has a more precise sense. It denotes mathematical correlations, universalizing the results of experiment, formulating in a general way a physico-mathematical "fact."

Law differs from theory in that the former is based on direct empirical evidence.[14] It is an experimental generalization. If we may speak of actually observed and measured correlations as "facts,"—a common practice as was mentioned above—then, "Laws merely represent facts, collect them, describe them in a formula. They may be true or false; and they are true so far, and only so far, as they do represent facts."[15] They are then simply a generalization of experimental results.[16] Theories, on the other hand, are not susceptible of direct experimental verification. They are meaningful suppositions, hypothetically entertained. By them, the scientist seeks to account for observed laws. But the laws do not depend, as is sometimes argued,[17] on the theories which render them intelligible. Kirkoff's law is true regardless of our theory of electric circuits. Hooke's law does not depend on the theoretical account we give of stresses and strains. Laws are empirically observed. They do not share the epistemological insecurity of physical theories.[18] The latter may come and go. The laws remain.[19]

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[12] Ritchie, *op. cit.*, p. 155.

[13] *Physics: the Elements*, Cambridge, 1920, pp. 47 ff.

[14] *Ibid.*, p. 130.

[15] Schmidt, K., "The Existential Status of Facts and Laws in Physics," *The Monist*, vol. 43, p. 162.

[16] Poincaré, H., *La Valeur, de la Science*, Paris, 1914, p. 142.

[17] Cf. Duhem, *op. cit.*, p. 254; also Sorel, G., "Les Preóccupations Métaphysique des Physiciens Modernes," *Revue de Métaphysique et Morale*, vol. 13, p. 871.

[18] Lindsay and Margenau, *op. cit.*, pp. 20-1.

[19] It need hardly be mentioned that the terminology of physicists is sometimes rather loose; sometimes the distinction between law and theory made

A physical law is formulated in a mathematical equation. The uniformity of association generalized by law can be best expressed in the exact, universalizing language of number. The differential equations which are the usual mode of expression are both concise in form and general in content. By mathematical development, we can make them explicitly applicable to the particular problems of the general case.[20] They depend, it was noted in the discussion of the procedure of measurement, on the function concept which expresses ratios between quantities, ratios that are yielded by experiment.[21] Stated in this convenient form of differential equations, the laws of physics thus enable us to calculate one quantity from our knowledge of another (or others) of which it is a function and to which it is related by a definite ratio. As Planck has noted, "Every statement is a physical law which expresses a fixed, inviolable valid correspondence between measurable physical magnitudes, a correspondence which permits one of these magnitudes to be calculated when the others are known through measurement."[22]

Planck's analysis is also important for its emphasis on the physical nature of the magnitudes involved. It is false or at least equivocal to say, as does Poincaré, that a law is a differential equation.[23] Equations are merely the formal way of expressing laws. The concepts of mathematical physics represent—or at least have reference to—physical phenomena. Unless, as was noted above, the material foundations of physics be kept in mind, physics will be emptied of the physical.

Ernst Mach, one of the seminal minds of the present-day epistemology of science, held that law is merely an economy of thought. Man tends to make things easy for himself.[24] Economy is of the

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above is not observed. In such cases, the inconsistency may be signalized by the philosopher. We are inclined, for example, to speak of the law of gravitation which is not a law in the strictest sense but partakes of a theory. Lindsay and Margenau, *ibid.*, pp. 22-23.

[20] Campbell, N. R., *Measurement and Calculation*, p. 218.

[21] Bridgman, P. W., *Dimensional Analysis*, p. 99.

[22] *Wege zur Physikalische Erkenntnis*, p. 148.

[23] "Une loi pour nous . . . en un mot, c'est une équation différéntielle," *op. cit.*, p. 174.

[24] *Die Mechanik in ihrer Entwicklung*, Leipzig, 1904, pp. 6-7.

very essence of science. Physics aims to make future experiments unnecessary to the extent in fact that laws are nothing but concise, economic formulae which enable man to live and to learn with as little effort as possible. The results of science could be attained without method. However, because of the brevity of life and as a convenience of memory, man finds it profitable to summarize his thoughts in the most economic way.[25] These summaries are scientific laws. So, at any rate, says Mach.

The counter argument is not against the view that laws economize thought, but that for Mach they seem to have no other references. As Aristotle noted, science deals with universals, i.e., with general principles common to many things. All men are scientists, in the word's widest sense, because they all desire to know. Knowledge by its very nature consists in the discovery of the one in the many, the general in the particular, or, in the language of modern physics, the discovery of the invariants in change. Economy is thus the natural result, it is indeed the concomitant, of knowledge. But it is not the only aim of science. It is not even the chief aim. We may distinguish also between the practical and the speculative sciences. The latter seeks knowledge for its own sake, the former for the sake of action. Mach does not make this distinction. The result is that he does not arrive at the true nature of knowledge. Hence, he does not grasp the relation of economy to science. Knowledge or science is a search after causes. When we reach causes, whether they be formal, final, material or efficient, we have economized thought. But we have done so only because a knowledge of causes yields (from the objective world) the one, the general, the invariant, in short, the universal with which science deals.

It is not sufficient to say that science is a mere economy of thought unless we are willing to recognize that there is an "economy" of being itself with which knowledge enters into correspondence. Otherwise, we must espouse nominalism or idealism. We are able to reach generalities in knowledge only because there are generalities in being which measures our knowledge. If we have not misinterpreted Mach, he does not accept this extreme view of

[25] *Ibid.*, p. 530.

idealism. But his concept of science as a mere economy of effort without further qualification can easily lead to subjectivism, as indeed it has since done among those who came under Mach's influence.

Interpreted in the fashion we have tried to indicate, the concept of law as a mental economy can find a real place in an organic philosophy of science. There is likewise room for Poincaré's philosophy of scientific laws if we search for the kernel of truth it contains and which, as Augustine said, can be found in every false system of philosophy.

Poincaré held that scientific laws, as we know them, are conventions. In this view, he was naturally influenced by his mathematical genius.[26] He perceived that there can be many valid systems of mathematics. Why do we adopt the Euclidian scheme in preference to other geometries? Because, he said, it is convenient. In mechanics, we find that our principles are not susceptible to direct experimental proof. We continue to use them out of convention. But this convention, he adds, is not completely arbitrary. Certain experiments show us why it would be suitable.[27] The conventions were accepted by the founders of the science who were led on by experiments to formulate them. The experiments justify the conventions.[28] The idea of force, for example, arises from a sense of effort; the concept of energy can be better appreciated by comparison with our own activity; the idea of mass can be grasped by the sensation of holding a heavy object. All these comparisons seem rather crude to the advanced knowledge of the scientist today. But still they lie somewhere beneath the surface of his general laws.

In order to reconcile being and knowledge and to avoid the extremes of anthropomorphism which Poincaré tries to eliminate

[26] Bavink, B., *Ergebnisse und Probleme der Naturwissenschaft,* Leipzig, 1933, p. 220.

[27] *Science et Hypothèse,* Paris, 1916, p. 163.

[28] "La loi d'accélération, lá régle de la composition des forces ne sont-elles donc que des conventions arbitraires? Conventions, oui; arbitraires, non; elles le seraient si on perdrait de vue les expériences qui ont conduit les fondateurs de la science à les adopter, et qui, si imparfaites qu'elles soient, suffisent pour les justifier. Il est bon que de temps en temps on ramène notre attention sur l'origine expérimentelle de ces conventions," *ibid.,* pp. 133-134.

and of cosmological agnosticism which conventionalism would logically lead him to adopt, the traditional notion of analogy may be applied. Granted that we have only a feeble knowledge of such elusive realities as force, energy, mass, and so on, still there is room for at least an analogical correspondence of phenomena and the psycho-physical cognition by which we grasp at first what the phenomena are and how they relate. The same parallel may be adopted in cases of other laws. It is this analogical correspondence which frees the so-called conventions from their purely arbitrary character. The mind is in contact with a real object, not merely with a mental figment. Poincaré himself seems to state this view in rather explicit terms, though his thought as a whole indicates that he did not appreciate the notion of analogy which he suggested. The scientist does not create the scientific fact out of nothing, he says. He derives it from the crude fact. Hence, he does not make a fact in a free and arbitrary way. His freedom is always limited by the material with which he works.[29] On his own testimony, Poincaré is not an agnostic. His concept of scientific law, which he traced back to its crude origins, can meet traditional philosophy through the notion of analogy. It is analogy that explains why the conventions are not arbitrary and why they are somehow in correspondence with reality.[30] The crude ideas of force, mass, and so on are not without deep meaning. Pure conventionalism would lead to skepticism and subjectivism.[31]

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[29] *La Valeur de la Science*, p. 232.

[30] "Wenn ich mir willkürlich Naturgesetze zurechtlege und dann auf Grund derselben Maschinen erfinde, so funktionieren diese nicht: auf Grund willkürlichen Annahmen kann ich nicht prophezeien, wass die gebauten Maschinen leisten oder ob sie überhaupt etwas leisten; denn die Natur richtet sich nicht nach willkürlichen Konventionen," Becher, E., *Weltgebäude, Weltgesetze, Weltentwicklung*, Berlin, 1915, p. 9. Becher's words are of such explosive force against subjectivism in science that it is difficult to see how he could maintain, as he did, that objectivity was a matter of only high probability.

[31] In mathematics only can we begin with conventions and still be within the limits even of common sense which knows that a fall in obedience to the law of gravity or a shock in obedience to laws of electricity are not simply experienced because it is conventional to do so but because there are realities that impose themselves on man rather than obey his wishes.

It is these two extremes that Karl Pearson in his attitude toward scientific laws comes to accept. As a positivist, he is clearly influenced by Mach in his belief that a law of nature is "a *résumé* in mental shorthand, which replaces for us a lengthy description of the sequences among our sense impressions."[32] But for him laws are not discoveries. They are inventions. "Law in the scientific sense is thus essentially a product of the human mind and has no meaning apart from man."[33]

The idealism of Pearson is contravened by both reason and experience. If scientific laws enable us to predict future phenomena, then they are not mere mental constructs. They are ontologically valid. Science is not an arbitrary fiction of the individual human mind. It begins in experience. Experience checks it. The fact that the empirical generalizations which we call the laws of nature are reliable guides to our knowledge of what future experiment will produce is evidence that they are not the caprice of man but have at least an analogical correspondence with a world that is truly objective."[34]

Pearson is a positivist in his odium metaphysicum.[35] He glorifies the scientific method as the sole legitimate organon. It is the virtue of the positivists to insist that scientific method be freed as far as possible from all extraneous ideas that warp the unity of the science.[36] It is their vice to belittle the vision, imagination, physical insight, and emotional warmth which are the tools of the original thinker in science. So also is it their vice in philosophy to take over the scientific method and to attempt its application, with the same rigorous purism, to all knowledge (and for all ethics too, whatever that term may mean to a positivist.) It is only in prob-

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[32] *The Grammar of Science*, London, 1900, pp. 80-87.

[33] *Ibid.*, p. 7.

[34] Bavink, *op. cit.*, p. 242.

[35] *Op. cit.*, pp. 14 ff.

[36] "Now, in Greek, it (method) literally means *a way, or path, of transit*. Hence, the first idea of Method is a progressive transition from one step in any course to another; and where the word Method is applied with reference to many such transitions in continuity, it necessarily implies a Principle of *UNITY WITH PROGRESSION*, Coleridge, S., *Coleridge's Treatise on Method*, ed., A. D. Snyder, London, 1934, p. 2. (Italics and capitals are in text.)

lems where the data are quantified, says the positivists, that we can expect to apply true science. The method of approaching the problems is to look for mathematical relations. If the positivists leave room for philosophy, it is to achieve a synthesis of the sciences or to perform the functions of logic in clarifying the meaning of scientific concepts.

Positivism renounces the search after causes. It insists on mathematizing knowledge through and through. The highest act of knowledge consists in measurement and the discovery of empirical law. Hypotheses must be eliminated altogether or be reduced to the mathematical background upon which numerical laws may be projected. As Comte wrote, "Every hypothesis, in order to be really capable of being judged, ought to bear upon the laws of phenomena exclusively and never on their mode of production."[37] Measurement alone is meaningful.

Avenarius, for example, proposed a philosophy of "pure experience" in which the function of the scientist is only to observe and report data.[38] There is no place here for hypotheses, for creative imagination, for theoretical ventures into the unseen world of physical reality which lies in back of the seen world that is mathematically described. For Petzoldt, such theories are works of art not of science.[39] Aliotta shares in these views.[40] Positivism has gained a strong foothold among both scientists and philosophers. It has a tremendous influence on the books and articles which philosophers in America have been publishing on both physics and philosophy. A nation like France has had a violent intellectual reaction against positivism under the impact of Bergson. Such a counter-movement has not yet begun in America. Here positivism still rules the field.

Positivism also reaches an extreme form in the so-called *Wiener Kreis* which wields great influence on present-day critics of science, as a review of contemporary books and periodicals amply bears

[37] *Cours de Philosophie Positiviste*, Paris, 1887, vol. 2, p. 312.
[38] *Kritik der reinen Erfahrung*, Leipzig, 1888, vol. 2, p. 352.
[39] *Einführung in die Philosophie der reinen Erfahrung*, Leipzig, 1900 (*passim*).
[40] Aliotta's philosophy is, however, a commingling of positivism, Kantianism, idealism, immanentism, and is not a doctrine of "pure experience."

witness. The doctrine of the Viennese school may be summed up in two principles: a) the physical sciences alone can attain valid knowledge of the real world; and b) the function of philosophy is an analysis of scientific concepts from the standpoint of logic with a view to clarifying their meaning.[41] Neurath views modern "scientism" as being antimetaphysical and empirical; it aims at a rigorous logic and at a mathematization of all the sciences.[42] Such are the characteristics of logical positivism as the movement has come to be known.

Logical positivism traces its origin back to Wittgenstein, who viewed all philosophy as a means of clarifying thought,[43] and to Bertrand Russell, who regards philosophy as logic.[44] It was developed by Wittgenstein's followers, chiefly Schlick and Carnap. Carnap, attempting to construct the concepts of all the empirical sciences by logical analysis, must, as Stebbing has shown, end in solipsism.[45]

For logical positivism, the meaning of a concept is its method of verification. The idea was advanced by Wittgenstein.[46] It undoubtedly expresses a truth that is interwoven in scientific method. It is because of the illicit transfer of this method to all fields of knowledge that logical positivism can find no place for philosophy except as logic and that Carnap's system, like the Cartesian doubt, must meet the fate of idealism and skepticism.[47] A sentence is

[41] Cf. Carnap, R., *The Logical Syntax of Language*, London, 1937, p. 279.
[42] *Le Développement du Cércle de Vienne et l'Avenir de l'Empirisme Logique*, Paris, 1935, p. 41.
[43] "Der Zweck der Philosophie ist die logische Klärung des Gedanken.
"Die Philosophie ist keine Lehre sondern eine Tätigkeit.
"Ein philosophisches Werk besteht wesentlich aus Erläuterungen.
"Des Resultat der Philosophie sind nicht 'philosophische Sätze' sondern das Klarwerden von Sätzen.
"Die Philosophie soll die Gedanken, sie sonst, gleichsam trüb und verschwommen sind klar machen und scharf abgrenzen," *Tractatus Logico-Philosophicus*, London, 1922, p. 76.
[44] "Philosophy, if what has been said is correct, becomes indistinguishable from logic," *Mysticism and Logic*, New York, 1918, pp. 111-112.
[45] *Logical Positivism and Analysis*, London, 1935 (*passim*).
[46] Cf. *supra*, n. 43.
[47] "In my opinion, Wittgenstein's conception of *verifiability* depends upon a serious equivocation with regard to the given, and hence, upon a muddle

meaningful only when it involves the rules by which it can be verified.[48] As Carnap says, the specification of an object consists in giving criteria of truth for those statements where the sign of the object occurs.[49]

The *Wiener Kreis* represents positivism in an extreme form, the death, so to speak, of the age of metaphysics in Comte's division of history and the complete absorption of philosophy by the positive sciences. It aims to unify all science on the basis of physics. It would make science itself a pointer-reading process like that prescribed by Avenarius and his school. "What is at all scientifically expressible," writes Neurath, "is no richer in fundamental relations than the symbols on a Morse tape which the telegrapher reads as they are sounded by his apparatus. In a sense, unified science is physics in its largest aspect, a tissue of laws expressing space-time linkages—let us call it: *Physicalism.*"[50]

Closely allied to the logical positivism of the Viennese School is the operationalism of Bridgman. Here, a concept is reduced to operations by which we measure. Bridgman writes, in describing his system, "In general, we mean by a concept nothing more than a set of operations; *the concept is synonymous with the corresponding set of operations.* If the concept is physical, as of length, the operations are actual physical operations; namely, those by which length is measured; or if the concept is mental, as of mathematical continuity, the operations are mental operations, namely those by which we determine whether a given aggregate of magnitudes is

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with regard to the notion of *direct experience* and content, as these are understood by the Logical Positivists. *This table* is not *an experience* of mine. Hence, in saying 'I perceive this table,' I am *not* saying 'I perceive an experience of mine.' Hence, in the case when I do perceive this table, *this table* is indirectly *given.*" Stebbing, L. S., *op. cit.*, p. 28. In a similar way, Carnap, in his *Logische Aufbau der Welt*, is unable to establish an intelligible relation between his "knowledge by acquaintance" and "knowledge by description." Cf. Russell, B., *The Problems of Philosophy*, London, 1932, pp. 72-92.

[48] Schlick, M., *Sur le Fondement de la Connaissance*, Paris, 1935, p. 37.

[49] *Der Logische Aufbau der Welt*, Berlin, 1928, p. 222.

[50] "Physicalism," *The Monist*, vol. 41, p. 620.

continuous."[51] Statements which cannot be tested by operationalism, Bridgman refers to as verbalisms and therefore meaningless.[52] Only what is determined by space-time instrumental relations has "physical reality." Since the existence of an objective world cannot at present be operationally proved, he urges that the possibility of its non-existence should inspire physicists to "make more provision for such a possibility in our thinking."[53] Bridgman does not use the label positivist, but his system puts him definitely in the company of the modern thinkers who exalt the empirical method as the only tool of genuine knowledge. He is a positivist in his opposition to "the metaphysical taint."[54]

In appraisal of the foregoing opinions, it must be affirmed that as far as positivism seeks to eliminate the obtrusion of the metaphysical on the purely formal aspects of physics, it commends itself to a genuine philosophy of the sciences. But the errors of the system seem to be: a) an over-insistence on the purely mathematical, reportorial nature of physics; b) the opinion that the scientific method alone is a legitimate technique for study and that philosophy, if it is valid at all, has nothing to say about the real world but only about logical entities.

Physics cannot free itself from the concept of a *physical* reality whose quantitative attributes are reduced to methematical law. Meyerson, though sometimes inclined to exaggerate the ontal significance of science as opposed to its empirical formality, has

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[51] Bridgman, P. W., *The Logic of Modern Physics*, p. 5; the author applies this view not only to empirical knowledge but to all cognition, *ibid.*, p. 32.

[52] Bridgman has attempted to exemplify his operationalism in *The Nature of Thermodynamics.* The reader may wonder, however, whether he has really carried it out to its logical conclusions. He admits, for example, in defining temperature, that "The starting point is the undoubted qualitative connection between the temperature concept and our crude physiological sensations of hot and cold," pp. 10-11. Quality as such is, however, not mensurable, and already, unless we misinterpret Bridgman's thought, we have the introduction of a non-operational element. Secondly, we may ask whether the physiological operations (sensations of hot and cold) constitute our temperature concept, according to Bridgman's canon, or whether the temperature concept is not the result of these operations. If this is true then operationalism fails.

[53] *The Nature of Thermodynamics*, p. 219.

[54] *Ibid.*, p. 216.

brought to the fore that physical science demands the concept of a reality, a thing.[55] "It is only when the locus of those measure-numbers is discovered in certain *qualitative* continuities, familiar enough in our daily activities that the predictive power of science becomes intelligible."[56] Physics is not only mathematics. It erects its mathematical laws upon a physical insight into an objective, physically existing world. If we trace back physico-mathematical theories, even those which are most highly mathematical, we find that in their origins they exact knowledge of phenomena in the real world which interact with one another and possess physical properties. Such theories demand also, in their application, insight into a physical situation. Even Bridgman has written, "Let anyone who maintains that there is nothing in nature except pointer-readings or coincidences engage to reproduce the situation that gave rise to the pointer-readings in terms only of the framework and the pointer-readings themselves."[57] Physics does not consist simply in an historical, reportorial account of mathematical properties. It demands vision, originality, intuition, discernment of causes, of new *physical* relations, visualizations of atoms, of electrons, of spin, and even—at least in an indirect way—of such supposedly unpicturable phenomena as space-curvature and wave-properties. It is only thus that the mathematical statements of modern physics can have physical sense and predict phenomena that we check with our senses in physical experiment. Otherwise we should become lost in a dim haze of mathematical symbols. Experiment would lose its meaning. Hypotheses are the work of keen minds which transcend reportorial observation in order to make their experiments intelligible. As Huxley has written, those "who refuse to go beyond facts rarely get as far as fact."[58] And Schroedinger has reached "the inevitable conclusion that we cannot close the door to the entry of subjective factors in determining our scientific policy and in giving a definite direction to our line of further advance."[59] Physics involves more than mere laws, mere declarations of fact.

[55] *De l'Explication dans les Sciences*, pp. 17-44; cf. *supra*, p. 8.
[56] Nagel, *op. cit.*, p. 74 (italics mine).
[57] *The Nature of Physical Theory*, Princeton, 1936.
[58] *Methods and Results*, New York, 1894, p. 62.
[59] *Op. cit.*, p. 87.

This is not to belittle the attempt of modern physics to purge itself of anthropomorphic elements. It is laudable to preserve the unity of the physicist's formal object.[60] But between anthropomorphism and a cosmic agnosticism of physical reality, there is the light of reasoning by analogy which genuine philosophy may submit to the modern physicist. Our ideas of force, of mass, of waves, and so on may not be projected into the exterior world exactly as we know them from our psycho-physical sensations. Yet they may have an analogical correspondent in the outer world. In opposing the so-called anthropomorphism of classical physics, contemporary scientists and philosophers are in danger of introducing an anthropomorphism of their own, based on the naive view of the world as a mere mathematical reality and a naive view of man as a completely passive observer.[61]

Positivism also goes to extremes in reducing all valid knowledge to that empaled by the scientific method. On this basis, scientific method, whether it be discussed in terms of logical positivism or operationalism, must be applied to establish its own validity. We are immediately caught in the contradiction of circular demonstration. The positivist opinion is based upon the unfounded belief that because the scientific method has achieved such great success in one field of knowledge it can qualify as the method of all valid learning. Positivism has swept on to its logical conclusion the doctrine long drifting through modern philosophy that the only reality is that of quantity and that the only knowledge is that of the senses.

Against this view, any genuine philosophy can reaffirm: a) the existence of a reality even in matter which is not mere quantity and which renders being intelligible—this is form; b) the existence in man of a supra-sensory, spiritual, intellectual power which can grasp this formal principle and which need not be governed, in its methods of knowing, by the quantitative methods of science. In fact, quantity as such does not make sense by itself. It is sheer

[60] ". . . so können wir kurz zusammenfassend sagen, die Signatur der ganzen bisherigen Entwicklung der theoretischen Physik ist eine Vereinheitlichung ihres Systems, welche erzielt ist durch eine gewisse Emanzipierung von der anthropomorphen Elementen, speziell den specifischen Sinnesempfindungen," Planck, M., *Wege zur Physikalischen Erkenntnis*, p. 5.
[61] Riezler, K., *op. cit.*, p. 73.

plurality of parts. Since, then, there are realities even in the material world which are higher than quantity; since man has powers of knowing in which the quantitative, scientific method is transcended, positivism as a cosmology, psychology, and theory of knowledge is incomplete. Positive science is complemented in its method of knowing by the method of philosophy and in its attitude toward phenomena by a philosophy of being.[62]

Logical positivism and operationalism much stand trial with positivism in general. Both may render a real service to physics in eliminating principles which are meaningless to the physico-mathematical structure of the science; in stressing clear definitions according to the physicist's canons of verification.[63] Secondly, though empirical physics must define its concepts in terms of methods of verification or in terms of operation, both logical positivism and operationalism have, if their systems are logically carried out, misconstrued the character of a concept and of definition which defines concepts. Both logically and psychologically, there must be a representation before empirical operations can be performed and scientific verifications carried out. Otherwise, knowledge is reduced to the sterile, senseless movement of operating on operations and of verifying verifications.

Psychologically this does not occur. Logically it would be an absurdity. We represent the *something* which is measured, verified, operated on. Without such awareness, there could be no science in the psychological sense.[64] As Planck has said, "The measurement gives no immediate results which have a meaning of their own."[65] In the psychological sense, we must therefore represent

[62] A detailed discussion of these two points lies much beyond the scope of the present work. The first question, the necessity of form, will be dealt with to some extent in the final chapter. For a defense of the second point, the reader is referred to Sheen, F. J., *The Philosophy of Science,* Milwaukee, 1934, *praesertim,* pp. 105-124; Rousselot, P., *L'Intellectualisme de Saint Thomas,* Paris, 1911; and Maritain, J., *Les Degrés du Savoir,* pp. 399-484.

[63] Maritain, J., "Science, Philosophy, and Faith," in *Science, Philosophy, and Religion: A Symposium,* New York, 1941, p. 170.

[64] "Unless one knew independently of the instrument what the instrument was designed to measure, it could never have been created," Benjamin, A. C., *An Introduction to the Philosophy of Science,* p. 116.

[65] *Where Is Science Going?* p. 95.

a *something* to be treated by the mathematical formality of mathematical physics.[66]

In the logical order, operationalism and logical positivism, if their premises are carried to the conclusion they suggest, do not conform to the rules of definition. They identify meaning with operation or with method of verification, in the last analysis. They leave no room, if their doctrines are followed to the letter, for anything but the circular process mentioned above—operating on operations. A logically sound definition based on measurement cannot be constructed without reference to what is being measured or verified, a reference that is grasped by intellectual insight, seeing "a whole that is articulated within itself."[67] Operationalism and logical positivism, considered as canons of verification in physics, must be grounded in a principle which is not mathematically, operationally, positivistically reducible. This principle in man grasps the ultimately non-quantitative but starkly real, objective, compelling relation between being and measurement. This is judgment, bridging the gulf between two ideas and binding them, by the reflective, bending-back of spirit on itself into a unity which is a polarized contrast to what physics treats, namely, quantity. We speak of things which are measured in this or that way; of things which are verified in this or that way. We grasp intellectually a relation which is, as a relation, not in the things but between them. The thing being measured is the genus and the particular measurement or verification forms the specific difference. Together they comprise a species. With such a restriction, logical positivism and operationalism are not only valid but highly desirable canons for the physicist. They insist that in defining the various objects of his study, he must because of the point of view (formal object) adopted by his science, define them in terms of measurement and verification.[68]

[66] As Bavink argues, against Carnap in particular, conceptualization (Begriffsbildung) and measurement (Messung) are not synonymous, *op. cit.,* pp. 218 ff.

[67] Cf. *supra,* p. 24.

[68] Stebbing writes: "We cannot clarify our thoughts by thinking about thinking, nor by thinking about logic. We have to think *about* what we *were* thinking about. . . . This investigation is not linguistic. We must first know

A corollary of fundamental importance may be drawn from the foregoing analysis. If intellectual insight, which is not mathematically reducible, should and does operate both in justifying the systems of operationalism and of logical positivism and in the actual use of the two systems, then it follows that there is a legitimate knowledge outside the empirical and the mathematical method used by physics. The scientific method cannot be, it is not in practice, the only tool of knowledge. There is a subject-matter in the material world which physics itself cannot grasp but which philosophy attains by intellectual analysis and synthesis. Once we have reached agreement on this point, we have come a long way toward the settlement of the disputed frontiers between physics and philosophy.

Law, it was seen, is an empirical generalization which expresses a relationship in the real world through the medium of mathematical functionality. With proper interpretations, Mach, Poincaré, and the positivists in general can find their place in a genuine, organic philosophy which recognizes that reality has many aspects and that the aspects studied by physics do not exhaust matter's knowability.

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what facts are the case before we can fruitfully employ analysis for the purpose of clarifying our thoughts about the world," *op. cit.*, p. 36. This statement is a telling one against the view that philosophy must confine itself to the domain of logic.

CHAPTER V

The Function of Theory

Physics aims to correlate phenomena. To achieve this end, it is necessary to discover the more fundamental properties that the phenomena have in common. We go beyond direct experience which, in itself, reveals to us differences in the phenomena more than their community. We employ the method of hypothesis, what Planck has called "the indispensable tool of every inductive investigation."[1]

Newton's often-quoted maxim, *Hypotheses non fingo,* does not in reality contradict Planck's view. It does not argue towards a pure empiricism in physics, any more than hypotheses indicate that the science is solely *a priori.* Newton was insistent on the empirical, inductive character of physics. But he did not overlook its hypothetical, deductive aspects. He perceived in principle and in practice that the two are parts of one procedure which is called the scientific method.

Newton's philosophy of method may be summarized in his own words: "But to derive two or three general principles of Motion from Phaenomena, and afterwards to tell us how the Properties and Actions of all corporeal Things follow from those manifest Principles, would be a very great step in Philosophy, though the Causes of those Principles were not yet discovered: And therefore I scruple not to propose the Principles of Motion above mention'd, they being of very general Extent and leave their Causes to be found out."[2] He conceived of scientific method as an analytico-synthetic movement which passes from particular effects to general causes,[3] then "explaining Phaenomena proceeding from

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[1] *Wege zur Physikalischen Erkenntnis,* p. 150.

[2] *Opticks,* London, 1931, Query 31, pp. 401-402; cf. Burke, H. R., "Sir Isaac Newton's Formal Conception of Scientific Method," *The New Scholasticism,* 10, 1936, pp. 93-115.

[3] *Op. cit.,* p. 404.

them and proving the Explanations."[4] Already we note that Newton's methodology is neither inductive nor deductive but a mixture of the two. As he writes in the *Principia,* "the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature and then from these forces to demonstrate the other phenomena."[5]

His four *Regulae Philosophandi* also stress the inductive character of scientific concepts. We are to confine our explanations to the fewest possible causes needed to explain appearances (the principle of economy); the same effects are to be attributed to the same causes; qualities which are found in all bodies studied in experiment and which do not admit of more or less, are to be taken as the universal properties of all bodies; propositions, attained by induction, are to be held as true until the contrary is proved, either by the refinement of measurements or by the discovery of exceptions.[6] In these rules Newton's logic seems to be almost purely inductive and empirical. Here he seems literally to carry out his belief that "hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy."[7]

But what did Newton mean by hypothesis? If we look to the way in which he arrived at his most noted contribution to physics, his principle of universal attraction among bodies—the law of gravitation as it is commonly called—we find him to be using hypothesis in the restricted sense which it assumes in the vocabulary of the modern physicist. Newton set out to find a gravitational attraction between bodies that would give rise by deduction to Kepler's third law: that the squares of the periods of revolution of the planets around the sun are proportional to the cubes of the radii of the respective orbits. Employing the equation for the centripetal force necessary to keep a planet moving in its orbit, Newton set this formula equal to his own formula that the attraction of sun and planet varied as the inverse xth power of their distance apart.

[4] *Ibid.,* p. 405.

[5] *Principia Mathematica Philosophiae Naturalis* (Eng. transl. by A. Motte, revised by F. Cajori), Berkeley, Cal., 1934, pp. xvii-xviii.

[6] *Ibid.,* pp. 398-400.

[7] *Ibid.,* p. 547.

Using Kepler's observations and solving for x, Newton discovered the law of inverse squares. But what he was using was what we now call a scientific hypothesis. He started out with the supposition that the *sun and the planet attracted each other* with a force proportional to the inverse xth power of the distance between them. After he discovered that his principle was checked experimentally by the comparison of the acceleration of the moon toward the earth with the acceleration of freely falling bodies at the earth's surface, he found that he could derive all three of Kepler's laws from his mathematical calculations. He then generalized his principle according to the third of his *Regulae Philosophandi.* Here also is a hypothetical element, the extrapolation of the so-called law of gravitation to include all bodies.[8]

De Morgan has summarized the above method in the following: "Newton, ready prepared with the mathematics of the subject, tried the fall of the moon towards the earth, away from her tangent, and found that, as compared with the fall of a stone, the law of the inverse square did not hold for the moon. He deduced the ellipse, he proceeded to deduce the effect of the disturbance of the sun upon the moon, upon the assumed theory of *universal* gravitation."[9]

The dictum *Hypotheses non fingo* does not propose a pure empiricism in scientific method. The author of it did not mean, it seems, to exclude genuine *scientific* hypotheses from research. What Newton rejected were hypotheses in the sense of mere unverifiable guesses or metaphysical speculations.[10] He wanted to keep empirical science empirical.[11] But he used hypotheses in the scientific meaning; "if he had put into practice his *Hypotheses non fingo* in the sense in which it has been taken, we would not possess the law

[8] Actually, Newton's law does not explain all gravitational phenomena, v.g., deviations in the path of the planet Mercury, and the theory of relativity is deemed a more universal principle than Newton's statement.

[9] For a summary of Newton's method cf. de Morgan, A., *A Budget of Paradoxes,* Chicago, 1915, vol. 1, p. 86; and Case, "Scientific Method as a Mental Operation," in *Lectures on the Method of Science,* ed. T. B. Strong, Oxford, 1906, p. 18.

[10] Stebbing, L. S., *A Modern Introduction to Logic,* p. 316.

[11] *Ibid.,* p. 315.

of universal gravitation."[12] Newton's principle is not a law in the sense of an empirical generalization but "is the fundamental hypothesis of the Newtonian theory of gravitation."[13] The founder of classical mechanics was thus a representative of scientific method, where a hypothesis is the "indispensable tool of every inductive investigation."

Hypotheses are suppositions to account for the data of experiment. A distinction might be made between hypothesis and theory on the grounds that the former is a simple conjecture which, when verified according to scientific canons, transforms into the latter. But we speak interchangeably of the quantum hypothesis or quantum theory, relativity hypothesis or relativity theory. A distinction might also be made on the basis of extension, hypothesis being directed to a particular fact or group of facts and theory being concerned with a whole class of phenomena.[14] In the former category would be included such suppositions as that of electron-spin; the Pauli exclusion-principle, and so on. But in practice no distinction is observed between the two concepts.

Divided according to matter, theories in physics are of two types: mechanical and mathematical.[15] Mechanical theories emphasize the physical suppositions made in order to account for experimental evidence. The atomic theory, the kinetic-molecular theory, the wave or corpuscular hypotheses of classical optics, and the electrodynamical theory developed by classical physics on the model of a hydrodynamical system—all these are so-called mechanical theories. Mathematical theories lay stress on numerical aspects of physical suppositions. As examples, we have Fourier's theory of heat-flow, Maxwell's electromagnetic theory, and in general the systems of quantum mechanics and of relativity theory (this latter hypothesis tries to geometrize reality). But the distinction between the two divisions is not as sharp as might seem at first glance. The mechanical theories were treated mathematically by classical physics, as classical statistics bear witness. The atomic theory was

[12] Naville, E., *op. cit.*, p. 199.

[13] Lindsay and Margenau, *op. cit.*, p. 23.

[14] Thus, Margenau, H., "The Problem of Physical Explanation," *The Monist*, vol. 39, pp. 343-4.

[15] Cf. Meyerson, E., *op. cit.*, p. 35; Bavink, B., *op. cit.*, p. 31.

projected by Dalton because of the striking mathematical relations shown in the laws of definite and of multiple proportions. On the other hand, the mathematical theories of contemporary physics derive ultimately from mechanical hypotheses. Their origin can be traced back to *physical* presuppositions. They depend for their application on the knowledge of a physical situation. Ultimately the difference between the two types of theory depends on where the emphasis is laid.

"Mathematical" theories are not laws. The physico-mathematical suppositions are designed to yield empirically observed laws by deduction. Though the derived conclusions may check with experimental evidence we cannot validate our suppositions unless we prove that they alone can account for the facts. The three statistical systems employed in modern physics make suppositions concerning the original distribution and interrelationships of the particles. From these postulates, deductions can be made which accord with experiment and form statistical laws. But we have not thereby validated our presuppositions as the only ones which could account for the facts. The Schroedinger equation, a mathematical theory of quantum mechanics, yields by deduction the generalizations of facts that we check experimentally. It is possible that a different mathematical supposition could likewise account for the laws empirically observed. While laws, derivatives of theories, are true because of direct experimental evidence, the theories themselves, even of the mathematical type, are only provisional.

Divided according to method, theories may be, as Rankine said, hypothetical or abstractive.[16] They are hypothetical when conceived prior to observed facts and tested by subsequent experiment. They are abstractive when drawn from the observed facts. This is somewhat akin to de Morgan's distinction between Bacon's method of using facts to construct theories and the method, actually practiced by scientists, of using facts to test theories.[17]

Actually, both methods are in vogue in modern physics. A true hypothesis is a combination of the two. Planck's quantum hypothesis arose by an abstractive procedure, to use Rankine's term.

[16] Rankine, W., *Miscellaneous Papers*, London, 1881, p. 210.

[17] *Op. cit.*, vol. 1, p. 88.

The wave theory of matter was advanced by de Broglie before having any immediate experimental evidence on which to work. Maxwell's electromagnetic theory of light was not verified until Hertz's experiments twenty years later. Yet no theory is completely *a priori* and suppositional and then later verified, as Whewell seemed to hold.[18] In a strict sense, it is always a "post-supposition," based on previous knowledge which it seeks to interpret.[19] In this sense, all theories are drawn from observational data. But once a theory has been conceived, its logical consequences are deduced and compared with experiment. Thus the facts are used to test the hypothesis which suggested them.[20] The theory which successfully interpreted the initial facts and predicts new ones is placed on a more solid footing.[21]

The empiricism of Bacon and Mill has been easier to describe in the classroom than to follow in the laboratory. Bacon left no room for hypotheses, for anticipations of nature. Mill admitted theories. But he misconceived their nature. A thorough-going empiricist, he viewed hypotheses as capable of becoming proven facts. He refused to regard as the truth-test of a theory its usefulness in "suggesting a line of investigation."[22] Here he is at variance with modern physics. His canons, because they leave no room for intellectual analysis and prescribe pure induction, are neither methods of discovery nor methods of proof. If we decide to let the facts speak for themselves, we can ask: what are the facts and which ones should we study?[23] The value of the Baconian method and of Mill's canons can be gauged by the fact that no scientist has ever used them. They are not methods of proof because their crudely empirical character does not permit the generalization of experimental fact which is a scientific law. If they are methods of discovery, then they pertain to the psychology of

[18] *A History of Scientific Ideas*, London, 1858, vol. 1, p. 29.

[19] d'Abro, A., *Evolution of Scientific Thought from Newton to Einstein*, p. 437.

[20] v. Hartman suggests that theories are intimated by facts to the mind of the physicist who then works backward to verify his ideas. *Philosophie des Unbewusstseins*, Berlin, 1874, p. 355; cf. also pp. 260-280.

[21] Case, T., *op. cit.*, p. 27; Naville, *op. cit.*, pp. 59-60.

[22] *A System of Logic*, London, 1895, p. 325.

[23] Cohen and Nagel, *op. cit.*, p. 201.

invention and not to demonstrative logic. In the strict sense, there are no rules for discovery, "the methods of investigation are many and lawless, whereas the methods of proof are one and law-abiding."[24] A discussion of the process of discovery, which is a psychological question, is at most of very indirect concern in appraising the logical significance of science. Psychologists themselves cannot give a satisfactory answer to the nature of genius, talent, the inventive activity of the mind which is involved in scientific discovery. Discovery is an art, not a science. "Francis Bacon, the official herald and announcer of the scientific method, totally misunderstood this quality in his charge. He imagined that the scientific method was itself as mechanical a thing as the picture of the universe which it produces by its efforts."[25] The scientist cannot explain his applied logic in passing from one premise to another and finally to a conclusion. He "proceeds largely by hunches and guesses, by trial and error, by intuition, by unconscious inferences and techniques based primarily on habit. The point is not that these are bad methods but that they are accepted uncritically."[26]

Demonstrative proof presupposes abstraction. It is based ultimately on propositions that are self-evident.[27] The predicate is seen to be contained within the subject by an act of apprehension.[28] Strict demonstration may therefore be said to proceed from a knowledge of essences in which the middle term is predicated of the minor and the major is predicated of the middle.[29]

Scientific method, measuring as it does only the metrical aspects of the material world, is closed off from a knowledge of essences. But it seeks admission to that world at a lower level. It has in fact proposed a substitute deductive system of its own—the method of hypothesis. A theory is "the practical equivalent of essence."[30] With this as a base, the scientist sets out to deduce "attributes."

There is a methodological parallel also in the scientist's process

[24] Dubs, H., *Rational Induction*, Chicago, 1930, p. viii.

[25] Needham, J., *op. cit.*, p. 81.

[26] Benjamin, A. C., *An Introduction to the Philosophy of Science*, p. 30.

[27] *In I Anal. Post.* 3.

[28] *Ibid.*, 1.

[29] Joseph, H., *An Introduction to Logic*, Oxford, 1916, pp. 308-309.

[30] Maritain, J., *op. cit.*, p. 67.

of arriving at a theory and the philosopher's apprehension of form. Though a theory may be suggested by facts of science and though a knowledge of essence may be derived from philosophical facts, in a genuine sense both are abstractions.[31] Both seek to provide a basis for facts observed, the philosopher in the world of rational necessity, the scientist in the world of theories. Scientific method is often referred to as inductive. But induction is only a step in the procedure of physics. The ladder is many-runged. To explain observed facts in the quantified world, it remains in the world of quantity and seeks quantified antecedents in time and in nature which account for the facts.[32] We deduce the effects from hypothetical causes. In philosophy, however, causes and essences are not supposed. They are abstracted by intellectual analysis of the data of sense. The result is not a mere seizure of quantified antecedents (either mechanical or mathematical). It is the grasp of intelligible form.

Accordingly, one of the fundamental marks of a hypothesis is that it permit deductions to be made from it. The wider the scope of the theory, the more firmly it is established. This is so because the more universal his hypothesis the more deeply the physicist seems to probe into the intimate and ultimate structure of the world of quantity—his substitute for the philosopher's concept of essence.[33] "Hypotheses are valuable for science," write Cohen and Nagel, "in proportion as they permit an organized deduction of consequences which are applicable to domains qualitatively different."[34]

Maxwell's theory united electromagnetism and optics. Quantum theory has been of far-reaching significance for modern physics because it has united the study of material particles and of radiation which, in the classical system, were considered as two separate and

[31] Naville, E., *op. cit.*, p. 208; Whitehead, A., *Principles of Natural Knowledge*, p. 108; Needham, J., *op. cit.*, p. 82; Westaway, F., *Science and Theology*, London, 1920, p. 252; Eddington, A., *The Philosophy of Physical Science*, p. 55; Planck, M., *Where Is Science Going?* pp. 86-87; Duhem, P., *op. cit.*, p. 77; Maritain, J., *op. cit.*, p. 49 note.

[32] Becher, E., *op. cit.*, p. 15.

[33] Ritchie, *op. cit.*, p. 59.

[34] *Op. cit.*, p. 283.

irreducible domains. A similar unifying purpose was served by Einstein's general theory of relativity. Newton had already generalized Kepler's laws of planetary motion into the principle of gravitation which laid the groundwork for a universal mechanics. Einstein generalized Newton's "law" itself. The synthesizing theory of relativity aims to combine gravitational, electrical, and magnetic phenomena into the all-embracing concept of the field theory in which matter and energy are equivalent. Quantum mechanics and relativity theory on the level of science thus strive after something more fundamental in nature than the bifurcated theories of classical physics.[35] The quantum and relativity theories thus provide a more remote aspect of essence in its substitute form. From these hypotheses, a greater number of deductions can be made than under the disparate theories of classical physics.[36] "Our experience up to date," says Einstein, "justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity."[37] This is only a modern paraphrase of Laplace's belief that in classical mechanics all motion could be reduced to a single mathematical equation. The aim of physics then is to attain the most fundamental mathematical relation, the most fundamental schema, the most remote quantities in the make-up of matter, from which all other facts can be deduced.[38]

In a sense, a theory never really dies in the history of science. It is merely corrected and transformed. Bohr did not abolish the Rutherford atom-model. He merely corrected it. The two-fluid theory of electricity has reappeared in the form of the electronic theory of positive and negative charges. The theory of photons did not abolish the wave-theory of light. It built a new system in

[35] Planck, M., *Physikalische Rundblicke*, Leipzig, 1922, p. 121; also, by the same author, *The Universe in the Light of Modern Physics*, London, 1921, p. 21.

[36] Dotterer, R., *Philosophy by Way of the Sciences*, p. 283; Lenzen, V. F., *The Nature of Physical Theory*, New York, 1931, p. 47.

[37] *On the Method of Theoretical Physics*, New York, 1933, p. 15.

[38] "An ideal shines in front of us, far ahead perhaps but irresistible, that the whole of our knowledge of the physical world may be unified into a single science which may be expressed in terms of geometrical or quasi-geometrical conceptions," Eddington, A., *The Nature of the Physical World*, p. 136.

which the undulatory hypothesis was included. Still less does the Einsteinian concept of gravitation, in ordinary space, abolish the Newtonian one. The reason for this is not far to seek. If the old theories were able to account for phenomena, their content must be embodied somehow in the new hypotheses that take their places.[39] Einstein writes that since the special theory of relativity grew out of the Maxwell-Lorentz account of electromagnetic phenomena, the arguments that support the latter also support the former.[40] This merely confirms the opinion above that science seeks by hypothesis after the most fundamental metrical character of matter. From it physics seeks to deduce all else.

The important function of deduction in the procedure of modern physics cannot ratify the description of scientific method as a purely inductive process. Bacon, Mill, Hume, and Herschel were not the philosophers of science, either from the philosophical point of view which rejects their methods on principle or from the scientific standpoint which rejects their method in practice. The method of modern physics is a mixed method which begins in experience, transcends experience to account for it, and after a deductive development of its abstracted suppositions through mathematical and logical inference, returns to experience to test its account and to gather fresh material for a repetition of the cycle.[41] Smart "looks upon induction and deduction not as two separate methods or techniques, but as two inseparable aspects of that process of reasoning which has as its main task the progressive deepening and extending of the knowledge we already possess."[42] Even in the statistical domain of modern physics, we find both inductive and deductive processes. Statistics is not a mere enumeration. Its actual development has resulted largely from a process of deductive reasoning.[43] The fundamental assumptions of quantum statistics are the hypotheses of equal *a priori* probabilities and of random a priori phases for quantum states. Though these assumptions are not amenable to direct testing, the mathematical laws to

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[39] Benjamin, A. C., *The Logical Structure of Science*, p. 224.
[40] *Relativity*, p. 58.
[41] Case, T., *op. cit.*, pp. 23-24.
[42] Smart, H. R., *The Logic of Science*, New York, 1931, p. 43.
[43] Tolman, R., *op. cit.*, p. 4.

which they give rise have been found to agree with experiment. The original assumptions have been validated for the scientist by the derived conclusions.

But if physics is not completely inductive, neither is it a completely deductive system. Hence, Descartes' logic has been modified by the historical direction of the science he helped to found.[44] Physics is not completely deductive, *a priori*. It must begin in the external world. It must return to it. If the facts do not fit the theories, it is the theories that must go, not the facts. The facts come first in the order of knowing; the theory is first in the order of being, hypothetically so in the sense discussed above.[45] If indeed theories precede facts according to de Morgan's dictum, this is only because the theories themselves have been constructed on a knowledge of previous facts. Deduction thus becomes in a sense a *via inventionis*. It reaches farther than pure induction and gives it birth. It is creative of new knowledge.[46] If a theory is true, then certain facts can be predicted from it. The prediction of phenomena, which physicists proclaim to be the sole purpose of a theory, constitutes the verification of the supposition.

Verification is a process of mensuration and observation to check the deductions from hypotheses. In direct opposition to Bacon's views, the modern physicist does not believe that he can interpret nature unless he can first anticipate her; nor that he has obeyed nature, unless he has first commanded her. Against Mill, he finds the great value of his theories in their ability to suggest new lines of investigation. He reverses Mill's suggestion of ascribing assumed laws to known causes. He does not follow the pure induction of the five canons. Herzfield's statement may be repeated here: "Understanding of a phenomenon means that we can reduce a large number of complex phenomena to a few simple principles which might enable us to predict ahead of time what will happen under given conditions in the future."[47]

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[44] Gilson, *Etude sur le Rôle de la Pensée Médiévale dans la Formation du Système Cartésien*, p. 180.
[45] Cf. *supra*, pp. 2 ff.
[46] Heisenberg, W., *op. cit.*, p. 17.
[47] "The Frontiers of Modern Physics and Philosophy," *Proc. Amer. Cath. Phil. Assoc.*, 1930, p. 40.

Verification is a return to the data of sense from the universe of reasoning. If the theory can forecast what data should occur in experiment, it is said to be verified. De Broglie's hypothesis of matter-waves predicted the results of the Davisson-Germer experiment which was conducted sometime later. The prediction was a potent argument in favor of the theory. The hypothesis of general relativity was placed on a far solider basis through the deflection of light observed during the solar eclipse of May 29, 1919 by the Joint Committee of the Royal and Royal Astronomical Societies. If physics is a study of matter as metrical, the chief function of theory is to serve as a guide to new experiments, new measurements, new metrical data.[48]

This predictive function of a theory is constantly emphasized by the scientists in their literature. The majority of the leading physicists seem agreed that their purpose is not to probe into the ontology of matter. It is merely to predict the results of experiment. Thus Dirac writes: "The only object of theoretical physics is to calculate results that can be compared with experiment."[49]

The Newtonian concept of gravitation was once accepted as a working principle: but Einstein's law of gravitation was better able to predict phenomena in stellar space. Newton's principle was called into question as a universal law. The undulatory theory of light was accepted by the 19th century because it was able to explain interference and diffraction-bands; but it was unable to predict all the facts concerning light, for example the photoelectric effect. Hence, the particle theory was revived. Considering light as both a particle and a wave, physicists have been able to predict phenomena verifiable by experiment. Until the day when the existing theory fails in its forecasting, physics will continue to employ it as a sound working principle.

The emphasis on theory as a mere instrument to predict the results of experiments has tended to the opinion that theoretical physics has nothing to say about the structure of the real, is concerned only with the forecasting of *laboratory* data. Dingler, for instance, writes that theoretical physics does not concern the real.

[48] Dingler, H., *op. cit.*, p. 175.

[49] Cited by Ruark and Urey, *Atoms, Molecules, and Quanta*, New York, 1933, p. 219.

Its purpose is a practical one and has been successfully tested. So long, he says, as theoretical physics stimulates experimental physics to new empirical findings, it has justified itself.[50] It has no connection with reality. This is the view of the idealist and positivist mentality. Against it, it must be affirmed that experiment and reality are not two different worlds. Though reality may be altered, sometimes in a radical way, by the very act of experiment and by its isolating character, the matter and energy involved are still integral parts of the quantified world to which nothing has been added or taken away. Even Bergson, who attacked science as an artifact, recognized that experiment was not completely artificial and arbitrary.[51]

Art does not add to the material constants of nature. It recombines them in new ways. An experiment is an event in the real world. It is real itself. If it is artificial, in the common meaning of that word, it is none the less natural in its efficient and material causes which are the only etiology which physics knows. A theory in physics not only predicts the results of experiments. It predicts real events. Though experiment may interfere with nature, it does not destroy or even diminish it. We may by proper corrections, argue back to a more or less exact knowledge of nature itself. Theories are projected to render experiments intelligible. Experiments, however, are part of reality. Even interference conveys important evidence concerning the object which is studied and which behaves in this or that way under alteration.[52]

The predictive function of theory as conceived by the contemporary physicist confirms the view that method in physics is a movement toward the deduction of facts from the substitute concept of essence (on the level of quantity) which is the middle term of scientific proof. As Plato and Aristotle noted, science is of universals. Universals are general truths because they are independent of time and space, which are the factors of change. The fact that physics seeks after principles that hold at various times and various places indicates that it is seeking after those fundamental, universal metrical characteristics of its metrical subject-matter.

[50] *Op. cit.*, pp. 37-38.

[51] *L'Evolution Créatrice*, p. 108.

[52] Benjamin, A. C., *An Introduction to the Philosophy of Science*, p. 105.

Such characteristics vindicate the scientific nature of physics itself, at least its tendency to be a science. Without unchanging, universal principles, there could be no science. Physics seeks order in the world. This quest is a struggle after the unifying notes which reduce into a system the multiplicity that our senses find in matter. If there were only individuals in the world, everything would be unique. There could be no comparative judgments. These acts of the mind are required to discover general relations among phenomena and thus to construct science. Positivism to the contrary, it is obvious that abstraction—the act by which the mind arrests quiddity or essence (in the wide sense of the words)—is an integral part of scientific method. It is only by this mental apprehension that we can rise above the mobile, existential world of sense cognition and grasp those characters of experience which are not peculiar to the passing moment. Such characters lie in the region of the universals where any science must make its home.

It is by no means an accident, then, in physics, which is a science, that the aim of theory should be the prediction of new phenomena and that verification should be only the test of its ability as a forecaster. If a theory is successful in its predictions, it is said to be verified. The ultimate significance of this attitude toward verification is that independence of time, place, and the accidental variations, introduced by the experimenter to determine more precisely the functionality involved, is in itself evidence that the theory reaches invariants, essential characteristics (as far as physics goes) of the material studied. It is these characteristics which enable the physicist to make general statements.

The world of theory is still hypothetical, even in its own order of quantified being *qua* quantified. This imposes obvious provisions on the general statements themselves. "Consequently," writes Nagel, "while every instance of the application of the theory is a test of the theory, the theory is formulated for *possible* as well as actual cases, and is not, therefore, completely verified in the latter."[53]

The theory is a working hypothesis which guides the scientist in his experiment. It can be modified whenever facts warrant,

[53] *Op. cit.*, p. 57.

precisely because of its hypothetical nature. In this sense, the world of theory is more a seeking than a finding, more a *tool* in Dewey's vocabulary,[54] than an ultimate end which is good in itself because it is true knowledge. Yet the representative character of a theory is not absent. For this reason, as we saw, theories are never completely abandoned. They are only revised and re-interpreted. The very fact that a theory can predict future phenomena proves that it has in some way represented the real world.[55] Maritain's distinction between the progress of science as *extensional* movement and that of philosophy as an *intension* or deepening of knowledge is appropriate.[56] Physics moves along the surface of the real. The wreckage of its old ideas scattered over its path is ample testimony to the fact that it is always seeking, seeking indeed to disprove its own hypotheses and find new ones which will, like their predecessors, be only provisionally entertained. As Lichtenberg has asserted, truth is the asymptote of research. Philosophy, on the other hand, moves downward and inward through the ontal structure of matter. It is still seeking and always will be; for the ultimate answers to man's questions will be found only in the pure being which is the God of traditional philosophy. But its search is on a different level where the ultimate answers have in some measure been found. Philosophical progress consists in exploring the ultimates by a deeper and deeper penetration. The empirical physics of Aristotle has long been abandoned. His philosophical physics (cosmology) has survived—a testimony of its contact with the real and not merely hypothetical world of causes and reasons where the spirit of man may enter.

The nature of hypothesis has now been defined sufficiently for present purposes. Theory has been located within the logical framework of physics. Having seen the relation of theory to physics, we may now probe further into the relation of theory to reality.

[54] "Logical ideas are like keys that are shaped with reference to the opening of a lock," *How We Think*, New York, 1933, p. 134.

[55] Duhem, *op. cit.*, p. 41.

[56] *Les Sept Leçons sur l'Etre*, Paris, pp. 6 ff.

CHAPTER VI

The Significance of Theory

If hypotheses are provisional, when do they become true?

Actually, forward-looking physicists do not attempt to transfer their physical speculations to the world of being as an absolute explanation. Contemporary physicists regard the so-called mechanical theories as anthropomorphic. They refuse to trace mathematical hypotheses to picturable counterparts in the real. The purpose of a theory is to predict, not to explain or picture. When it can no longer forecast the future, it will be amended accordingly. As a theory, then, it is never considered to be true in the genuine sense of the word. Though it is said to be verified by its prediction, this verification is no inexorable conformity of intellect and reality. "If the new facts agree with the theory," Herzfeld writes, "that does not give *logical* proof for it; that would only be the case if there existed two alternatives."[1]

This provisional acceptance of hypotheses was noted by Aquinas. He laid down principles of method on this point which provide rich material for a critical study of modern physics. An insufficient way of explaining a phenomenon, he writes, is to posit a cause that is merely *consistent* with the effect in question. This, as he added, was the case in the astronomical system of his day in its theory of eccentrics and epicycles.[2] But this method of explaining, Aquinas adds, is not sufficient for true proof. It is possible that the observed facts can be accounted for by a different hypothesis.[3]

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[1] "The Role of Theory in Modern Physics," *The New Scholasticism*, 8 (1934), p. 320.

[2] Duhem remarks that in the Aristotelian astronomy of homocentric spheres, we discern a genuine example of scientific method as it is practiced today and a genuine case of scientific theory. Beginning with a number of simple geometrical principles, deductions are made to explain observed data, *Le Système du Monde*, Paris, 1913-1917, vol. 1, p. 128.

[3] "Ad aliquam rem dupliciter inducitur ratio. Uno modo ad probandum sufficienter aliquam radicem. . . . Alio modo inducitur ratio non quae sufficienter probat radicem, sed quae radici iam positae ostendat congruere consequentes effectus; sicut in astrologia ponitur ratio excentricorum et epicyclorum, ex hoc quod, hac positione facta, possunt salvari apparentia sensibilia circa motus caelestes: non tamen ratio haec sufficienter probans, quia etiam forte alia positione facta salvari possent," I, 32, 1 and 2.

We cannot say that the theory which we have conceived is the only one which can "save the facts" in question.[4]

The definition of truth as the conformity of intellect and reality is the traditional correspondence-doctrine. If physics does not claim such conformity for its theories, as Herzfeld and others declare, it nevertheless has its own criterion. From an inspection of the science itself, it would seem that this criterion is coherence. Aquinas suggests this fact in stating that a hypothetical explanation "shows that the effects deduced are congruous with an already posed principle."[3] The antecedent need only be consistent with the effect. It need not be its true cause.

The coherence doctrine of truth is the legacy of Hegel. It has been applied by its advocates to the whole of knowledge. "Truth in its essential nature," says Joachim, "is that systematic coherence which is the character of a significant whole."[5] And F. H. Bradley writes: "Thought essentially consists in the separation of the 'what' from the 'that.' It may be said to accept this dissolution as its effective principle. Thus it renounces all attempt to *make* fact, and it confines itself to content. But by embracing this separation, and by urging this independent development to its extreme, thought indirectly endeavors to restore the broken whole."[6] The function of knowledge is to systematize.[7] Truth consists in an "arrangement of ideas, self-consistent and complete."[8] Contradiction indicates that we have not found this harmonious system. We must redistribute the discrepancies "in a wider arrangement."[9] In

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[4] "Licet enim talibus suppositionibus facta appareant salvari, non tamen oportet dicere has suppositiones esse veras, quia forte secundum aliquem modum nondum ab hominibus comprehensum apparentia circa stellas salvantur," *In De Coelo et Mundo*, II, 17.

[5] *The Nature of Truth*, Oxford, 1939, p. 76.

[6] *Appearance and Reality*, London, 1893, p. 360.

[7] Joachim, *op. cit.*, p. 69.

[8] Bradley, *op. cit.*, p. 360.

[9] *Ibid.*, p. 303.

Joachim's judgment, scientific statements take their meaning from their place in the system.[10]

As in the case of physical science, knowledge is always seeking, never finding, pausing in its path, but never attaining its goals. "Truth," says Bradley, "is the predication of such content as, when predicated, is harmonious, and removed inconsistency and with it unrest. But because the given reality is never consistent, thought is compelled to take the road of indefinite expansion."[11]

Though profoundly thought through and beautifully written, Bradley's philosophy, neither at its origin nor in its development, admits of contact with being. In a genuinely Platonic sense, thought, for Bradley, comes to grips with only the *whatness* of things. It does not grasp, as realism compels, *existent* essences, i.e. essences with reference to existence. For Bradley, the mind stretches out toward a "self-consistent, all-inclusive" system of content. For a realist, like Aquinas, since the mind is measured by its object, truth is a relation between the intellect and that which *is*. A genuine philosophy does not dwarf itself to a sheer knowledge of essences or "content." Not that which is thinkable is true but that which *is*, and the mind lays hold of truth, elaborates a system of knowledge, only in so far as it brings itself into conformity with this being that is.

The coherence doctrine of truth is not without its commendation. What is disastrous is to take it as the definition of truth itself. Consistency, though not the essence of logical truth, is its consequence. When our thought contradicts itself, we are sure that we have not yet brought our minds and being into conformity. Being is not self-contradictory. So inconsistency in thought goads us onward to "get at the truth." In inductive mathematics, the coherence doctrine is fundamental since the law of contradiction is the only law that holds in this domain. Hence, there can be, for instance, any kind of non-Euclidian geometry alongside the Euclidian one so long as it is not contradictory in itself. Lastly, the coherence doctrine of truth applies as a formal standard of physical theories, *qua* theories. When Aquinas talked about the congruity of cause

[10] *Op. cit.*, p. 95 and p. 102.
[11] *Op. cit.*, p. 165.

and effect in physical science, he was using a synonym for "coherence."

A theory does not probe down to existent causes. It seeks after content, what a thing is, without reaching fact, whether it is. Physics finds a cause that is consistent with the effects and then weaves this concept into the "self-consistent, all-inclusive whole." To see this, one has only to take the pulse of modern physics. In nineteenth-century physics, the corpuscular and undulatory theories of light were opposed. Quantum theory has made the two cohere by redistributing the "discrepancies into wider arrangement,"[12] Anomalies in the classical theory of measuring motion were transcended by the theory of relativity. Classical theory was not abandoned when quantum and relativity mechanics arrived. According to the Bohr correspondence principle, quantum theory becomes identical with classical physics when the energy involved in a problem is great enough. Likewise, relativity mechanics comes into play only when velocities approach that of light. Classical physics was not thrown out. It was restricted to a special case of other systems as physics, expanding its knowledge, entered the domains outside ordinary space. Such other systems, quantum and relativity mechanics, were discovered to be more inclusive,[13] more self-consistent, more unitary in system than classical physics which is now confined to ordinary space. When present-day theories no longer meet these demands, new ones will be projected that will be more consistent with observed data and that will "save the phenomena." We see here why theories, once well-tested, are never completely supplanted by new ones. Their content is incorporated in some way with the new system which is more coherent with experimental effects.[14] Though genuine philosophy cannot apply the coherence theory of truth to the whole of knowledge, it can nevertheless find room for such a doctrine in formally qualifying sci-

[12] Cf. *infra*, p. 163.
[13] "The test which I advocate is that of a whole of knowledge as wide and as consistent as may be. In speaking of system, I mean the union of these two aspects, and this is the sense and the only sense in which I am defending coherence," Bradley, F. H., *Essays on Truth and Reality*, London, 1914, p. 202.
[14] Weyl, H., *Mind and Nature*, Philadelphia, 1934, p. 46.

entific hypotheses. Their purpose is not truly to explain in the full and formal sense of the word. It is only to "save the phenomena."

The pragmatic theory of truth plays a role in physics as a test. Theories aim to predict phenomena, and "the truth or falsity of an idea and the question of whether it has definite meaning is relatively unimportant: what matters is that it shall give rise to useful work."[15] If one theory is able to predict observable data where others fail, it is accepted, according to scientific canons, as being in accord with the facts. The doctrine of judging truth by its practical consequences developed in America in the pragmatism of Peirce and James and in the instrumentalism of Dewey. Like the coherence doctrine, it was extended to the whole of knowledge. This was a false generalization. It overlooked the distinction between the useful and the good in itself, between the absolute reality which must be served and the relative reality that must serve it. But like the doctrine of coherence, pragmatism has a licit field of operation in the logic of scientific theory.

"The full reality of truth," James wrote, ". . . is always some process of verification, in which the abstract property of connecting ideas with objects is workingly embodied."[16] James insisted that an idea must work. Like a scientific theory, it must work in the concrete, directed toward definite objects.[17] "My thesis is that knowing here is *made* by the ambulation through the intervening experiences. If the idea led us nowhere, or *from* the object instead of towards it, could we talk at all of its having any cognitive quality."[18]

Dewey is closer to the scientific method by his own words. He declares that "scientific method is the only authentic means at our command for getting at the significance of our every day experiences in the world in which we live. It means that scientific method provides a working pattern of the way in which and the conditions under which experiences are used to lead ever onward and

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[15] Planck, M., *The Philosophy of Physics*, New York, 1936, p. 106.

[16] James, W., *The Meaning of Truth*, New York, 1910, pp. 202-203.

[17] "The trueness of an idea must mean *something definite in it that determines its tendency to work* and indeed towards this object rather than towards that," *ibid.*, p. 174.

[18] *Ibid.*, p. 141.

outward."[19] Knowledge is an instrument, the value of which is to be gauged by the work it performs. It must relate to concrete experiences. Its worth is determined entirely by its practical results.

Though the philosophy of pragmatism has not survived the attacks of its critics and though the pragmatic method cannot be extended to the whole of knowledge, it is the legitimate—indeed the only final test of a theory in physics. Such a test, agreement with experiment, must necessarily be a *practical* matter. Newton tested his principle of universal gravitation by comparing the phenomenon of a falling stone with the concrete behavior of the moon. Hertz confirmed Maxwell's views on the electromagnetic character of light by an indirect measurement of the waves shown to be propagated by his oscillator. In short, physics is an empirical science. Its viewpoints can include only those realities which have a practical, empirical meaning. Ideas that do not have practical, empirical, mensurable consequences are meaningless in physics. In speaking of physics, it can be affirmed with Margenau: "Constructs possess no generic properties which make them good or bad, acceptable or inacceptable; they derive their validity entirely from the success they achieve in providing satisfactory physical explanations. In the sense, then, that validity and hence reality depends on *function,* my view is essentially pragmatic."[20] Theories have meaning only in their reference to the practical experiments which they predict and account for.[21] The objection to the pragmatic theory of truth lies in extending it to the whole knowledge, where practical importance is an effect of truth, not its cause and criterion. In the scope of physics, pragmatism is a legitimate test

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[19] *Experience and Education,* New York, 1938, pp. 111-112.

[20] Margenau, H., "Metaphysical Elements in Physics," *Reviews of Modern Physics,* 1941, p. 184.

[21] Restricting our statements to physical science, we may say with Dewey that "ideas or hypotheses are tested by the consequences which they produce when they are acted upon. This fact means that the consequences of action must be carefully and discriminately observed. Activity that is not checked by observation of what follows from it may be temporarily enjoyed. But intellectually it leads nowhere. It does not provide knowledge about the situations in which action occurs nor does it lead to clarification and expansion of ideas," *op. cit.,* p. 110; cf. also *The Quest for Certainty,* New York, 1929, p. 109.